## Linear Algebra (MATH 3333 - 04) Spring 2011 Homework 4

Due: Fri. Mar. 4, start of class

**Instructions:** Please read the homework policies and guidelines posted on the course webpage. You may **not** use a calculator (or computer). Make sure to write your name, course and section numbers in the top right corner of your solution set, as well as the assignment number on top. Please staple your homework. Sections and exercises refer to the exercises in the required course text.

## Conceptual Questions (not to be turned in)

Go over Exam 1 and make sure you understand all the problems.

## Written Assignment

Total: 100 points

Each problem is worth 10 points unless stated otherwise.

Section 4.1: 20, 21, 22, 23

Section 1.6: 20

**Problem A.** (30 pts) Consider the linear transformation given by  $T = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$ . (i) Determine the density of the density of

- (i) Determine the domain and range this linear transformation.
- (ii) Determine its image.

(iii) Determine the solutions to  $T\begin{pmatrix} x\\ y\\ z \end{pmatrix} = \begin{pmatrix} 0\\ 0\\ 0 \end{pmatrix}$ , i.e., the set of points that T maps to the origin.

**Problem B.** Show the sphere  $S^2 = \{(x, y, z) | x^2 + y^2 + z^2 = 1\}$  is not the image of any linear transformation  $T: \mathbb{R}^m \to \mathbb{R}^3.$ 

**Problem C.** Consider the planes in  $\mathbb{R}^3$  given by y - z = 1 and x + y + z = 0. They intersect in a line  $\ell$ . Write  $\ell$  in the form  $\ell = {\mathbf{u} + t\mathbf{v} : t \in \mathbb{R}}$  for some vectors  $\mathbf{u}$  and  $\mathbf{v}$  in  $\mathbb{R}^3$ .