Linear Algebra (MATH 3333 - 04) Spring 2011
Homework 11

Due: Fri. May 6, start of class

Instructions: Please read the homework policies and guidelines posted on the course webpage. You may **not** use a calculator (or computer). Make sure to write your name, course and section numbers in the top right corner of your solution set, as well as the assignment number on top. Please staple your homework. Sections and exercises refer to the exercises in the required course text.

Reading

Sections 7.1, 7.2

Conceptual Questions (not to be turned in)

1. What does it mean for a matrix to be diagonalizable?

2. What do the eigenvectors and eigenvalues tell you about the geometry of a linear transformation?

Written Assignment

Total: 100 points

Each problem is worth 10 points, except where noted.

Section 7.1: 7 (20 pts)

Section 7.2: 8, 10 (20 pts), 19

Problem A. Let $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$.

(i) Show A is not diagonalizable.
(ii) Describe what A does geometrically.

Problem B. (15 pts) Let $A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$.

(i) Find the (complex) eigenvalues and corresponding eigenvectors of A.
(ii) Diagonalize A (i.e., write $A = PDP^{-1}$ where D is diagonal)
(iii) Use (ii) to show $A^n = \begin{pmatrix} \cos n\theta & -\sin n\theta \\ \sin n\theta & \cos n\theta \end{pmatrix}$. (It may be convenient to use de Moivre's identity $e^{i\theta} = \cos\theta + i\sin\theta$, though this is not necessary.)

Problem C. (15 pts) Determine the matrix which reflect about $z = x + y$ in $\mathbb{R}^3$. (Hint: first write down the matrix with respect to a convenient basis.)