NEW COMPARISON THEOREMS IN RIEMANNIAN GEOMETRY

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Abstract. We construct and use solutions, subsolutions, and supersolutions of differential equations as catalysts to link the hypotheses on radial curvature of a complete n-manifold M to the conclusions on the analysis or geometry of quadratic forms and second order differential operators. In particular, we prove Hessian Comparison Theorems and Laplacian Comparison Theorems on M, generalizing the work of Greene and Wu.: If the radial curvature K of M satisfies $-\frac{a^2}{c^2+r^2} \le K(r) \le \frac{b^2}{c^2+r^2}$ on $D(x_0)$ where $0 \le a^2$, $0 \le b^2 \le \frac{1}{4}$, $0 \le c^2$, and $D(x_0) = M \setminus \mathrm{Cut}(x_0)$, then

$$\frac{1+\sqrt{1-4b^2}}{2r}\Big(g - dr \otimes dr\Big) \le \mathrm{Hess}(r) \le \frac{1+\sqrt{1+4a^2}}{2r}\Big(g - dr \otimes dr\Big)$$

on $D(x_0)$, in the sense of quadratic forms, and

$$(n-1)\frac{1+\sqrt{1-4b^2}}{2r} \le \Delta r \le (n-1)\frac{1+\sqrt{1+4a^2}}{2r}$$

holds pointwise on $D(x_0)$, and $\Delta r \le (n-1)\frac{1+\sqrt{1+4a^2}}{2r}$ holds weakly on M. A volume comparison theorem is also given.

This is based on my joint research work with Yingbo Han, Yibin Ren and Shihshu Walter Wei, and further work with Shihshu Walter Wei.