Immersed Minimal Surfaces, Orientable or Nonorientable

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Abstract

Minimal surfaces in a Riemannian manifold M^n are surfaces which are stationary for area: the first variation of area vanishes. In this talk we focus on the"false branch point problem" for minimal surfaces which are of a prescribed topological type, either orientable or nonorientable. Jesse Douglas in 1939 proved the existence of a minimal surface of a given topological type having smallest area among surfaces of that type, mapping the boundary homeomorphically to a given disjoint union Γ of Jordan curves, **assuming** that surfaces of lower topological type with boundary Γ have strictly larger area. We shall show that such a mapping is *free of false branch points.* In the case $n = 3$ of codimension one, it follows that f is an **immersion** on the interior of Σ.

In all cases, whether or not the Douglas hypothesis holds, any area-minimizing mapping $f : \Sigma \to M^3$ defines an immersed surface with smallest area, bounded by the given configuration of curves, possibly a surface of lower topological type.