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WARPED PRODUCTS OF METRIC SPACES

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Abstract. The best context for warped products is with geodesic metric spaces. While most published works which use warped products are for Riemannian manifolds with a positive warping function, greater flexibility and utility are obtained by the generalization to metric spaces and allowing the warping function to have zeros. This is illustrated by three settings.

•**Potential Mechanics on Metric Spaces.** The simplest examples of warped products are surfaces of revolution. For those the geodesics are specified by Clairaut's Theorem describing the geodesics in terms of the angles they make with the latitudes. But there is another interpretation: it gives the concepts of conservation of energy and potential wells for motion in a one-dimensional space. Clairaut's Theorem holds for very general warped products and consequently can be used to define mechanics for a potential function on the base space. It is unnecessary to have notions of covariant derivatives and acceleration of curves!

•**Metrization of Topological Constructions.** One of the major achievements of the last century was the use of metric geometry (the Thurston Geometrization Conjecture and Perelman's proof of it) to prove the Poincaré 3-sphere conjecture. So it seems important to use special metrics to study topological spaces. The topological constructions of cones, suspensions, and joins have many applications; they have very special metric versions which are warped products. For example, the metric suspension of a constant-curvature sphere is again a constant-curvature sphere, for which it is essential to let the warping function have zeros.

•**Warped Products with Curvature Bounds (above or below).** We have proved a characterization of a warped product with a curvature bound in terms of curvature bounds on the base, the fiber, and a convexity condition on the warping function.