Math 2433 Homework #9
Solutions

Section 12.10

Use Table 1 to find the Maclaurin Series of the given function.

32. \( f(x) = e^x + 2e^{-x} \).
Solution: The series for \( 2e^{-x} \) is \( \sum \frac{(-x)^n}{n!} = \sum \frac{(-1)^n2x^n}{n!} \). Hence the Maclaurin series for \( f(x) \) is
\[
\sum (1 + 2(-1)^n) \frac{x^n}{n!}.
\]

34. \( f(x) = x^2 \tan^{-1}(x^3) \).
Solution: The Maclaurin series is
\[
x^2 \sum (-1)^n \frac{(x^3)^{2n+1}}{2n+1} = \sum (-1)^n \frac{x^{6n+5}}{2n+1}.
\]

47. \( \int x \cos(x^3) \, dx \).
Solution: We have
\[
\int x \cos(x^3) \, dx = \int x \sum (-1)^n \frac{(x^3)^{2n}}{(2n)!} \, dx = \int \sum (-1)^n \frac{x^{6n+1}}{(2n)!} \, dx = C + \sum (-1)^n \frac{x^{6n+2}}{(6n+2)(2n)!}.
\]

48. \( \int \frac{e^x - 1}{x} \, dx \).
Solution: We have
\[
e^x - 1 = \sum_{n=0}^{\infty} \frac{x^n}{n!} - 1 = \left(1 + x + \frac{x^2}{2!} + \cdots\right) - 1 = \sum_{n=1}^{\infty} \frac{x^n}{n!}.
\]
Thus,
\[
\int \frac{e^x - 1}{x} \, dx = \int \sum_{n=1}^{\infty} \frac{x^{n-1}}{n!} \, dx = C + \sum_{n=1}^{\infty} \frac{x^n}{n(n!)}.
\]

Section 13.1

4. What are the projections of the point \((2, 3, 5)\) on the \(x - y\), \(y - z\) and \(x - z\) planes?

Draw a rectangular box with the origin and \((2, 3, 5)\) as opposite vertices and with its faces parallel to the coordinate planes. Label all vertices of the box. Find the length of
the diagonal of the box.

**Solution:** The projections on the $x - y$, $y - z$ and $x - z$ planes are $(2, 3, 0)$, $(0, 3, 5)$ and $(2, 0, 5)$ respectively. I will leave the graphing part to you.

6.

(a) What does the equation $x = 4$ represent in $\mathbb{R}^2$. What does is represent in $\mathbb{R}^3$?

(b) What does the equation $y = 3$ represent in $\mathbb{R}^3$? What does $z = 5$ represent? What does the pair of equations $y = 3$ and $z = 5$ represent? In other words, describe the set of point $(x, y, z)$ such that $y = 3$ and $z = 5$.

**Solution:**

(a) $x = 4$ represents a line parallel to the $y$ axis intersecting the $x$ axis at $(4, 0)$. The same equation in $\mathbb{R}^3$ represents a plane parallel to the $y - z$ plane passing through $(4, 0, 0)$.

(b) The equation $y = 3$ represents a plane parallel to the $x - z$ plane passing through $(0, 3, 0)$. The equation $z = 5$ represents a plane parallel to the $x - y$ plane passing through $(0, 0, 5)$. The pair of equations together represents a line parallel to the $x$ axis and passing through $(0, 3, 5)$.

Find the lengths of the sides of the triangle $PQR$. Is it a right angled triangle?

8. $P(2, -1, 0), Q(4, 1, 1)$ and $R(4, -5, 4)$. **Solution:** We calculate

\[
|PQ| = \sqrt{(4 - 2)^2 + (1 + 1)^2 + 1^2} = 3.
\]

\[
|QR| = \sqrt{0 + (-6)^2 + 3^2} = \sqrt{45}
\]

\[
|PR| = \sqrt{(2)^2 + (-4)^2 + (4)^2} = 6.
\]

Since $|PQ|^2 + |PR|^2 = |QR|^2$, this is a right-angled triangle.

11. Find an equation of the sphere with center $(1, -4, 3)$ and radius 5. What is the intersection of this sphere with the $x - z$ plane?

**Solution:** The required equation is

\[
(x - 1)^2 + (y + 4)^2 + (z - 3)^2 = 25.
\]

The intersection with the $x - z$ plane is given by substituting $y = 0$ in the above equation. The equation representing the intersection is then

\[
(x - 1)^2 + (0 + 4)^2 + (z - 3)^2 = 25
\]
that is, 

\[(x - 1)^2 + (z - 3)^2 = 9\]

which is a circle with center \((1, 3)\) with radius 3.