Syllabus for August, 2001 Qualifying Exam in Topology

1. Fundamentals
   a. topology on a set, basis and subbasis for a topology
   b. subspace topology, product topology, quotient topology
   c. continuity, uniform continuity, homeomorphisms, quotient maps
   d. closure, interior, limit points, convergence of sequences
   e. metric spaces

2. Connectedness
   a. connectedness, local connectedness, components
   b. path-connectedness, local path-connectedness, path components
   c. preservation of connectedness under continuous maps, unions, products, etc.

3. Countability and Separation Axioms
   a. first countable, second countable, separable
   b. Hausdorff, regular, completely regular, normal
   c. Urysohn’s Lemma, Tietze Extension Theorem

4. Compactness
   a. compactness, local compactness, compactification
   b. limit point compactness, sequential compactness, Lebesgue numbers
   c. preservation of compactness under continuous maps, unions, products, etc.
   d. Tychonoff’s Theorem

5. Paracompactness
   a. open covers, paracompactness
   b. existence of a partition of unity subordinate to an open cover (“subordinate to” means the same as “dominated by”)

6. Complete Metric Spaces, Function Spaces
   a. completeness, completion, total boundedness
   b. Baire spaces
   c. compact-open topology on function spaces
   d. evaluation map, exponential correspondence

7. Fundamental Groups and Covering Space Theory
   a. homotopy of maps, homotopy of paths, fundamental group, Van Kampen’s Theorem
   b. homotopy equivalence, contractibility, deformation retraction
   c. covering spaces, path lifting, regular covering spaces

8. Examples
   a. $S^1, \overline{S^1}$
   b. lower limit topology, cofinite topologies, Cantor sets
   c. countable and uncountable products of $[0, 1], \mathbb{R}$
   d. manifolds, surfaces

References:

*Topology: A First Course* by J. Munkres
Chapter 1 of *Algebraic Topology* by Allen Hatcher, available at [http://math.cornell.edu/~hatcher/#ATI](http://math.cornell.edu/~hatcher/#ATI)