1. Prove: A closed map is a quotient map.

2. Prove: Let $X$ be a compact space, $U$ be an open covering of $X$. Then there exists a partition of unity dominated by $U$.

3. Let $(X, d)$ be a metric space.
Prove: If $X$ is complete and totally bounded (for every $\epsilon > 0$, there exists a finite covering of $X$ by $\epsilon$-balls), then $X$ is sequentially compact.
Give an example showing totally bounded in the statement (1) cannot be replaced by bounded.

4. Using only the definition of “compactness”, prove:
The closed interval $I = [0, 1] \subset \mathbb{R}$ is compact.

5. A subspace $S$ of $X$ is called a retract of $X$ if there exists a continuous map $r : X \to S$ such that $r \circ i$ is homotopic to $1_S$. Prove: The equator $S^1$ of the sphere $S^2$ is not a retract of $S^2$.

6. Let $T$ be the torus, $M$ be a closed disk. Form a connected sum $X = T \# M$. [That is, remove open disks $D_1 \subset T$ and $D_2 \subset M$, and glue a cylinder $S^1 \times I$ by homeomorphisms $S^1 \times \{0\} \xrightarrow{\sim} \partial D_1$ and $S^1 \times \{1\} \xrightarrow{\sim} \partial D_2$, where $\partial$ means the boundary]. Describe $X$ homotopically, and calculate the fundamental group of $X$. 