1. Prove or disprove:
   (a) If every subgroup of a group $G$ is normal, then $G$ is abelian.
   (b) If $D$ is PID then so is $D[x]$.
   (c) If $E/K$ and $K/F$ are finite Galois extensions then $E/F$ is also Galois.

2. Let $p$ be a prime number and $G$ be a group with $|G| = p^n$.
   Show:
   (a) The center of $G$ is non-trivial.
   (b) For each $k \leq n$ $G$ has a normal subgroup $H$ of order $p^k$.

3. Prove:
   (a) If $G$ is a group of order 12 then $G$ is not simple.
   (b) If $G$ is a group of order $p^2q$, where $p$ and $q$ are distinct primes, then $G$ is not simple.

4. If $I$ and $J$ are ideals of a ring $R$ and $I + J = R$, prove that $R/I \cap J$ is isomorphic to $R/I \oplus R/J$.

5. Show that an element $a$ of a commutative ring $A$ is nilpotent (i.e., $a^k = 0$ for some positive integer $k$) if and only if $a$ is contained in every prime ideal of $A$.

6. If $M$ is a finitely generated module over a Noetherian ring and $f : M \to M$ is an epimorphism, prove that $f$ is an automorphism.

7. Classify, up to similarity, all $3 \times 3$ matrices $T$ satisfying $T^3 = T$ over an arbitrary field $F$. 
8. Prove:

(a) If $G$ is a finite non-cyclic abelian group then there is a positive integer $k < |G|$ such that $g^k = e$ ($e$ is the identity of $G$), for every $g \in G$.

(b) Every finite multiplicative subgroup of the group of non-zero elements of a field is cyclic.

9. Let $E$ be the splitting field of $x^4 - x^2 + 2$ over $\mathbb{Q}$. Find all intermediate fields $K$ between $E$ and $\mathbb{Q}$.

10. Let $E$ be a finite extension of $F_q$ (the finite field with $q$ elements). Show that $E$ is Galois over $F_q$ and $Gal(E/F_q)$ is cyclic.