
Math 6393: Lie Groups and Lie Algebras II

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— 7th Assignment —
due April 22, 2008

22. Let \mathfrak{g} be a complex semisimple Lie algebra, let α be a root, and let $\mathfrak{s}^\alpha = \langle X_\alpha, Y_\alpha, H_\alpha \rangle$ as in class. Show that the elements

$$\begin{aligned} X_\alpha - Y_\alpha, \\ X_\alpha + Y_\alpha + iH_\alpha, \\ X_\alpha + Y_\alpha - iH_\alpha \end{aligned}$$

are eigenvectors for the operator $\text{ad}_{X_\alpha} - \text{ad}_{Y_\alpha}$. Use this to compute the quantity

$$\exp\left[\frac{\pi}{2}(\text{ad}_{X_\alpha} - \text{ad}_{Y_\alpha})\right](H_\alpha).$$

23. In problems 19, 20 and 21 you computed the roots of the Lie algebras $\mathfrak{sp}(4, \mathbb{C})$, $\mathfrak{so}(4, \mathbb{C})$ and $\mathfrak{so}(5, \mathbb{C})$, considered as elements of \mathfrak{h}^* . On each of these Lie algebras, consider the inner product

$$\langle X, Y \rangle = \text{tr}({}^t\bar{X}Y).$$

Use this inner product to identify \mathfrak{h} and \mathfrak{h}^* . Calculate the roots as elements of \mathfrak{h} . Also calculate all the co-roots.