CONVEXITY 2 SPRING 2000

HOMEWORK 4

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1. Let Q be an $(n-1)$-polytope in $\mathbb{E}^n$ with $o \in \mathrm{relint} Q$ and let P be the bipyramid over Q defined by $P = \mathrm{conv}(I \cup Q)$ where I is the line segment joining $\pm e_n$, the unit vectors orthogonal to $\mathrm{aff} Q$. Describe the polar body P^*.

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2. A 3-polytope is said to be simple if there are precisley three edges containing each vertex. Let P be a simple 3-polytope and let p_n denote the number of facets of P which are n-gons $(n = 3, 4, \dots)$. Prove that

$$\sum_{n \geqslant 3} (6-n) p_n = 12.$$

Use similar techniques to show that all 3-polytopes (not just those that are simple) must have at least one n-gonal facet with $n \leqslant 5$.

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3. Lay, Question 23.3

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4. Lay, Question 23.5

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5. Lay, Question 23.7