

CONVEXITY II

SPRING 2000

HOMEWORK III

1. Lay, Question 20.1

2. Lay, Question 20.2

3. Lay, Question 20.8

4. Let X^4 be a four dimensional crosspolytope.

a) How many vertices does X^4 have?

b) How many facets does X^4 have? Describe the nature of each of these facets.

5. Let M be the moment curve in \mathbb{E}^4 , that is $M = \{(t, t^2, t^3, t^4) \in \mathbb{E}^4 : t \in \mathbb{R}\}$. Let V be any finite subset of M and put $P = \text{conv}V$. Show that, if W is a subset of V comprising exactly four (distinct) points, then W is affinely independent. Deduce that every facet of P is a tetrahedron. Prove that every two points of V are vertices of P which are joined by an edge of P . Polytopes with this property are said to be neighbourly. Note that, in dimension 3, the only neighbourly polytopes are tetrahedra.