MATH 2513-001 **QUIZ IV** **NAME:** SOLUTIONS

Thursday, November 19, 2009

Q1]. . . Define what it means for a set A to be *countable*.

A set A is said to be countable if A is finite or if there exists a bijection $\mathbb{Z} \to A$.

Define what it means for two sets A and B to have the same cardinality.

Sets A and B have the same cardinality (written $|A| = |B|$) if there exists a bijection $A \to B$.

Say whether each of the following sets are countable or uncountable.

(1) $\mathbb{Q}$.
Countable. From class notes — similar to proof that $\mathbb{Z} \times \mathbb{Z}$ is countable. (Example 18 from Cardinality handout).

(2) $\mathbb{R}$.
Uncountable. From class notes — Cantor diagonalization argument. (Theorem 22 from Cardinality handout).

(3) The set of irrational numbers.
Uncountable. Since $\mathbb{Q}$ is countable, $\mathbb{R}$ is uncountable, and the union of two countable sets is countable. (Example 18, Theorem 22 and Example 19(a) from Cardinality handout).

(4) The set of all points in the cartesian plane.
Uncountable. Since it contains a copy of $\mathbb{R}$, eg. the x-axis, and subsets of countable sets are countable. (Theorems 22 and 20 from Cardinality handout).

(5) The set $\mathbb{R}^{\mathbb{R}}$ of all functions from $\mathbb{R}$ to $\mathbb{R}$.
Uncountable. Since it contains a copy of $\mathbb{R}$ as a subset, eg. $\{\chi_{\{x\}} \mid x \in \mathbb{R}\}$ is a subset of $\mathbb{R}^{\mathbb{R}}$, and subsets of countable sets are countable. (Theorems 22 and 20 from Cardinality handout).