

# 2010 Oklahoma PDE Workshop

## Title and Abstract

### **Deformable Multi-modal Image Registration by Maximizing Renyi's Statistical Dependence Measure**

Yunmei Chen  
University of Florida

Abstract:

A novel variational model for deformable multi-modal image registration is presented in this work. As an alternative to the models based on maximizing mutual information, the Renyi's statistical dependence measure of two random variables is proposed as a measure of the goodness of matching in our objective functional. The proposed model does not require an estimation of the continuous joint probability density function, which is sensitive to the quantization of the intensities. Instead, it only deals with observed independent samples. Moreover, the theory for reproducing kernel Hilbert spaces is used to simplify the computation. Experimental results with comparisons are provided to show the effectiveness of the model.

### **Numerical Prediction of Intense Local Weather: A Grand Challenge for PDEs, Uniqueness and Deterministic Predictability**

Kelvin Drogemeier  
University of Oklahoma

### **Axial Symmetry of Some Entire Solutions of Nonlinear Elliptic Equations**

Changfeng Gui  
University of Connecticut

Abstract:

In this talk, I will present some recent results on the axial symmetry of certain entire solutions which are anisotropic. The type of solutions includes stationary solutions for nonlinear Schrodinger equation, saddle solutions and traveling wave solutions for Allen-Cahn equations.

### **Asymptotic behavior of solutions to the $\sigma_k$ -Yamabe equation near isolated singularities**

Zhengchao Han  
Rutgers University

Abstract:

$\sigma_k$ -Yamabe equations are conformally invariant equations generalizing the classical Yamabe equation. YanYan Li proved earlier that an admissible solution with an isolated singularity at 0 in  $\mathbb{R}^n$  to the  $\sigma_k$ -Yamabe equation is asymptotically radially symmetric. In this joint work with YanYan Li and Eduardo Teixeira, we prove that such a solution is asymptotic to a radial solution to the same equation on  $\mathbb{R}^n \setminus \{0\}$ . These results generalize earlier pioneering work in this direction on the classical Yamabe equation by Caffarelli, Gidas, and Spruck.

## Continuous Harmonic Maps and Relaxed Energy

Robert Hardt  
Rice University

Abstract:

This is a talk on various works from the late eighties to early nineties and some more recent work on special classes of harmonic maps. The still open, motivating question is whether any degree zero, smooth map,  $g : \mathbb{S}^2 \rightarrow \mathbb{S}^2$ , of the 2 sphere to itself admits a smooth harmonic map extension,  $u : \mathbb{B}^3 \rightarrow \mathbb{S}^2$ , to the 3 ball.

## The Local Isometric Embedding Problem

Marcus Khuri  
SUNY Stony Brooks

Abstract: We give an overview of the problem of realizing Riemannian manifolds, locally, in Euclidean spaces. We will also present recent joint work with Qing Han on this topic.

## Stability of Some Fluid Type Problems

Congming Li  
University of Colorado

Abstract:

This talk will focus on the investigation of special structures related to the study of the dynamic stability of the 3D incompressible Euler and Navier-Stokes equations. Some special models have been studied. One simple model with global stability is used to construct global solutions with large initial value. We will also present some derivations of other models and the stability of these models.

## **Refined approximation of Landau-de Gennes energy minimizers**

Luc Nguyen  
Princeton University

Abstract:

In the context of nematic liquid crystal, it is of interest to see how Oseen-Frank (OF) energy minimizers can be used to approximate Landau-de Gennes (LdG) energy minimizers, which depends on a small parameter  $L$ . The simplest approach is that

$$\text{LdG minimizers} = \text{OF minimizers} + \text{correction}, (*)$$

where the correction term is “small” for small  $L$ . Because of the failure of OF energy minimizer in predicting “line defects” it is possibly desirable to have a better understanding of the correction term. We show that the correction term is of size  $L$  so that (\*) becomes

$$\text{LdG minimizers} = \text{OF minimizers} + F_1 L + o(L)$$

in an appropriate sense. Joint work with Arghir Zarnescu.

## **Diffusion and Spatial Inhomogeneity**

Wei-Ming Ni  
University of Minnesota

Abstract:

In this talk I will describe some of the recent progress on the effect of diffusion in Lotka-Volterra competition systems in heterogeneous environments.

## **A FAST ALGORITHM FOR EULER'S ELASTICA MODEL USING AUGMENTED LAGRANGIAN METHOD**

Xuecheng Tai  
University of Bergen, Norway

Abstract:

Minimization of functionals related to Euler's elastica energy has a wide range of applications in computer vision and image processing.

An issue is that a high order nonlinear partial differential equation (PDE) needs to be solved and the conventional algorithm usually takes high computational cost. In this paper, we propose a fast and efficient numerical algorithm to solve minimization problems related to the Euler's elastica energy and show applications to variational image denoising, image inpainting, and image zooming. We reformulate the minimization problem as a constrained minimization problem, followed by an operator splitting method and relaxation. The proposed constrained minimization problem is solved by using an augmented Lagrangian approach. Numerical tests on real and synthetic cases are supplied to demonstrate the efficiency of our method.

Joint with JOOYOUNG HAHN and GINMO JASON CHUNG

## **Space of the Ricci flows**

Bing Wang

Princeton University

Under the non-collapsing condition, we show that Ricci flows with bounded scalar curvature, bounded half dimensional curvature integral norm have weak compactness property. This weak compactness property has applications in the convergence of Kahler Ricci flows on Fano manifolds and the moduli space of Ricci solitons.

## **Well-posedness for geometric evolution equation with rough initial data**

Changyou Wang

University of Kentucky

Abstract:

In this talk, I will discuss the well-posedness question for two classes of evolution equations, namely harmonic map heat flow and the liquid crystal flow, and show that both of them are locally (or globally) well-posed for initial data in BMO space (or small BMO norm).

## **A variation problem for isometric embeddings into the Minkowski space**

Mu-Tao Wang

Columbia University

Abstract:

I shall discuss a variation problem arising from the study of quasilocal energy in general relativity. For a given spacelike 2-surface in spacetime, the Euler-Lagrange equation for quasilocal energy is the isometric embedding equation into the Minkowski space coupled with a fourth order elliptic equation for the time function. The solution of this system gives the ground configuration in GR. In joint work with PoNing Chen and Shing-Tung Yau, we solve this system in the important cases of large and small sphere limits.

## **On the bottom of spectrum of certain complete Kahler manifolds**

Xiaodong Wang  
Michigan State University

Abstract:

The bottom of spectrum is an important invariant for a complete Riemannian manifold. We are interested in both the upper and lower bounds assuming a negative lower bound on the Ricci curvature. I will first discuss some results in the Riemannian case. Then I will focus on the Kahler case. We will formulate a conjecture and discuss some partial results. This is a joint work with Song-Ying Li at UC Irvine.

## **Exotic Capillary Tubes**

Henry Wente  
Toledo University

Abstract:

In contrast to the standard capillary tube, an exotic capillary tube is a rotationally symmetric tube of variable cross section which if positioned correctly in a vessel of fluid possesses a continuum of equilibrium configurations. The controlling variables are the capillary constant and the contact angle of the fluid with the solid wall. Lowering the tube slightly from its natural position causes the tube to completely fill up while raising the tube slightly forces the tube to drain out. Other surprising consequences follow. Possible real world applications will also be discussed.

## **On the Wuff isoperimetric deficit**

Jiazu Zhou  
Southwest University

Abstract:

In this talk, we are going to discuss the Wuff isoperimetric deficit upper and lower limits. We also like to discuss the Gage-type isoperimetric inequalities and the Ros-type isoperimetric inequalities.

# **Application in Ecology**

Yiqi Luo

University of Oklahoma