Nowcasting Rotating Storms

Mathematical Geo-Science research group

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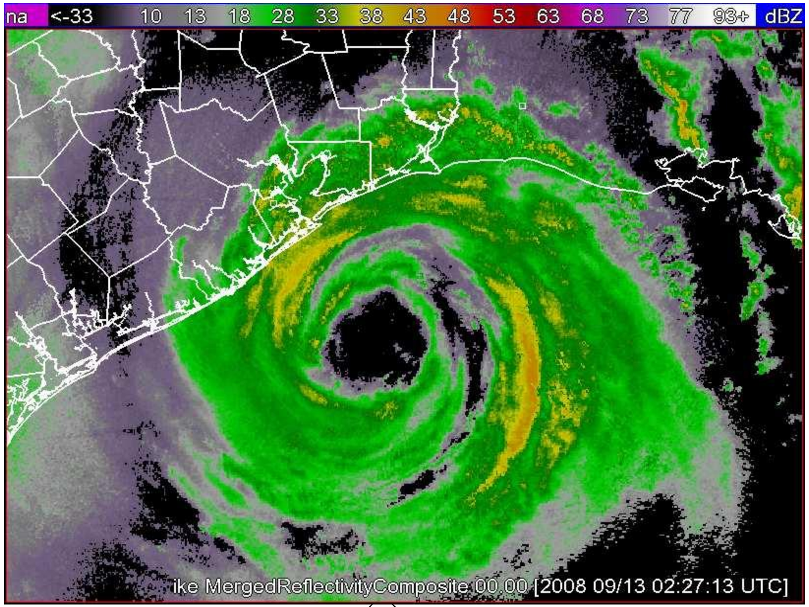
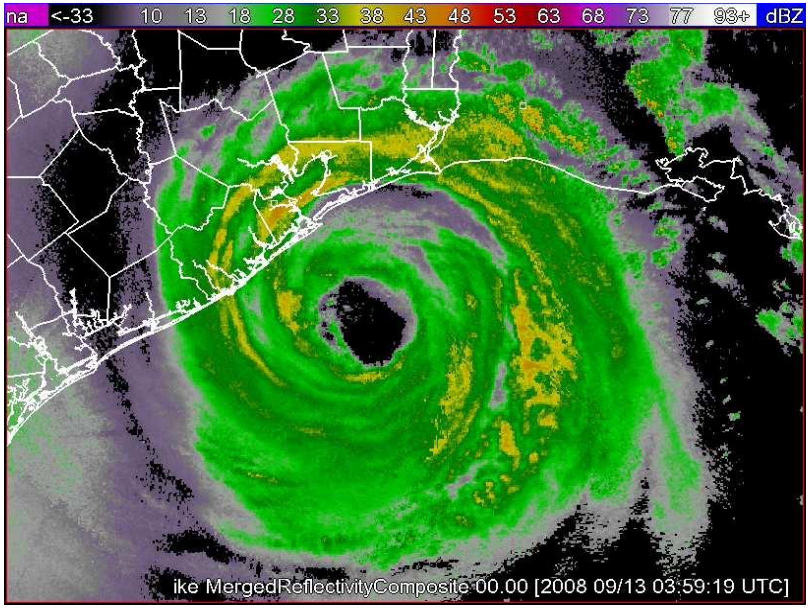
Rotating tropical storms



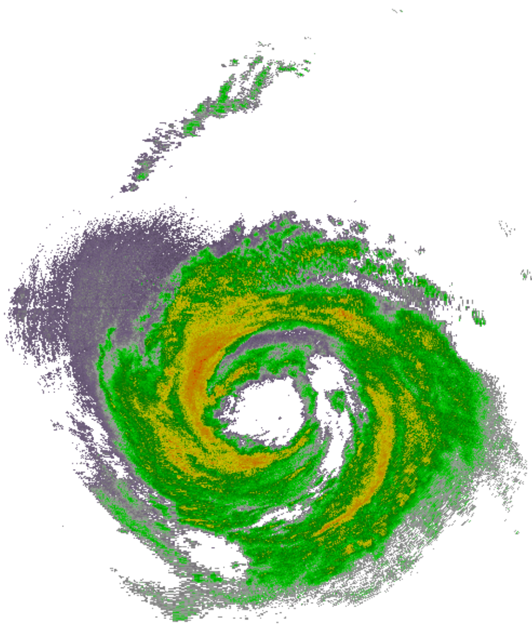
(a)

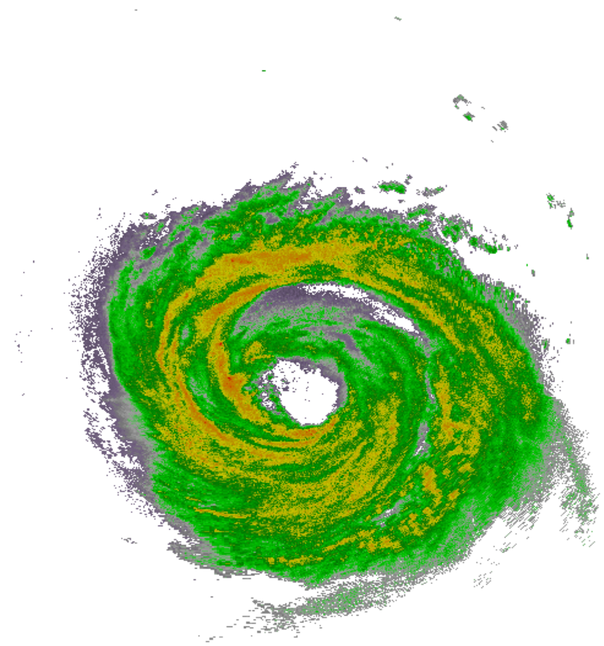


(b)

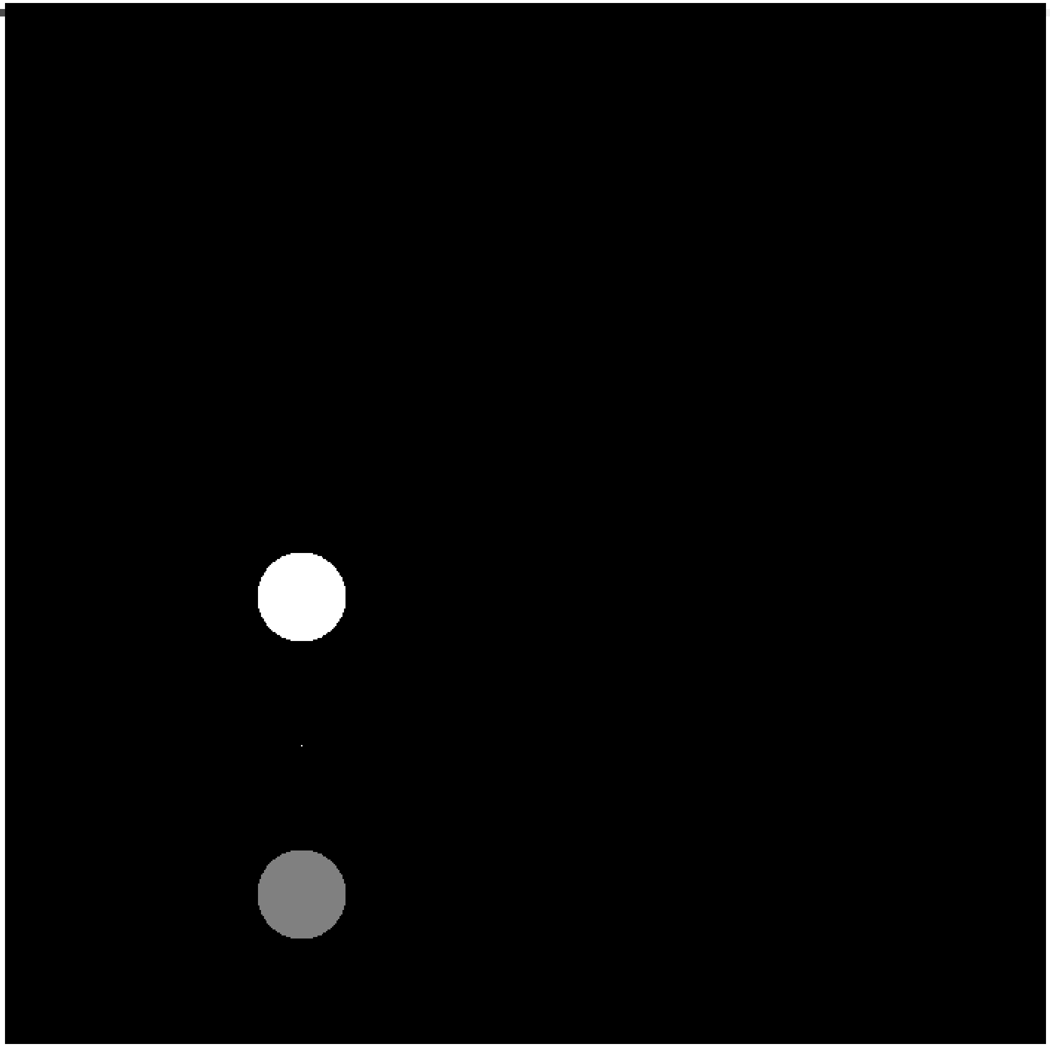
Hurricane Ike makes a landing at Houston, TX, Sept. 13, 2008.

Linear extrapolation?

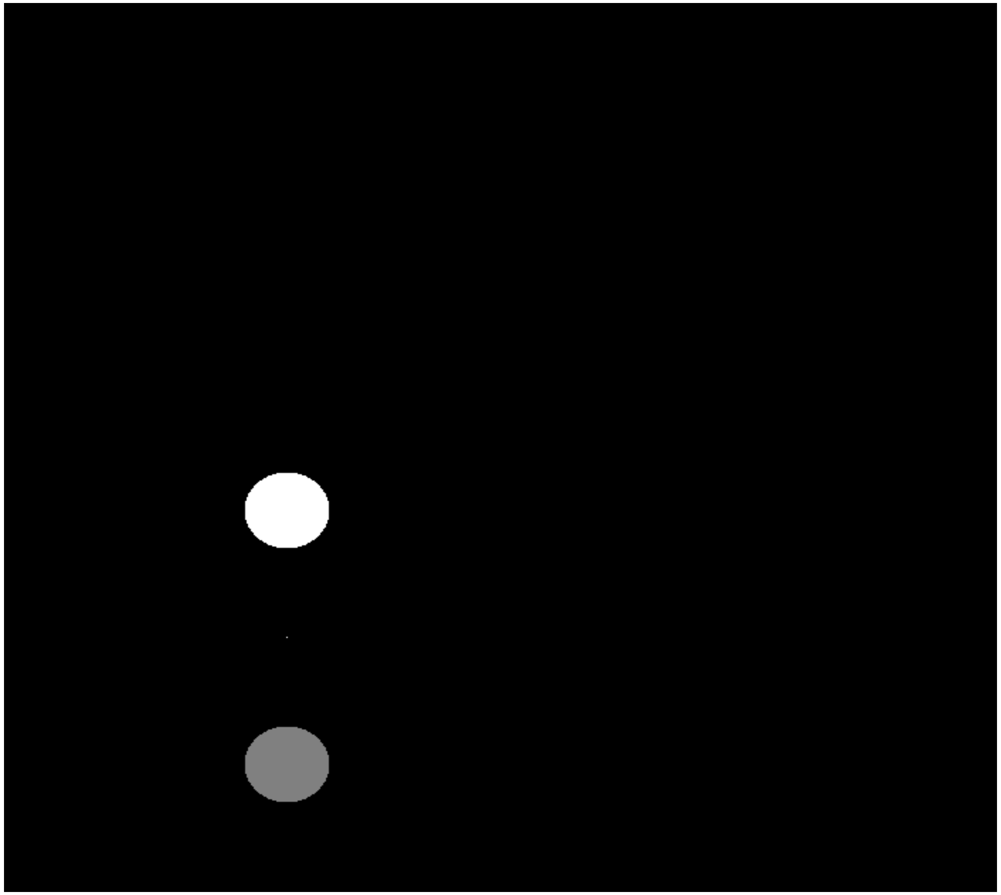




Synthetic rigid motion



Tracking motion via linear extrapolation



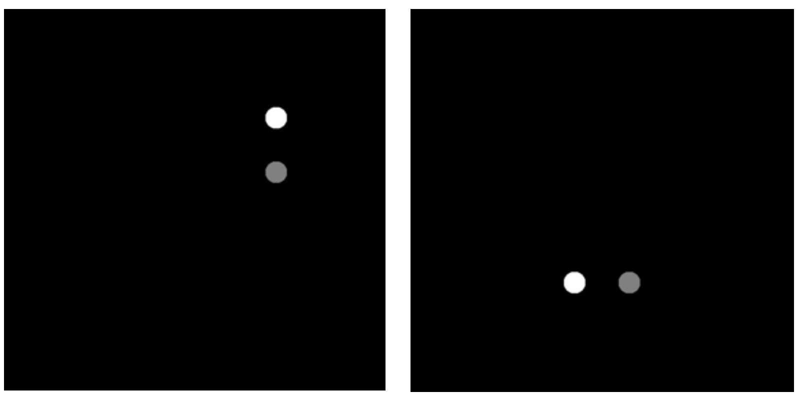
Difficulty

- Hard to capture rotating and translation simultaneously from remotely sensed data. Very few results concerning the rotating in nowcasting (short-term forecasting).

Assumption: Rigid motions.

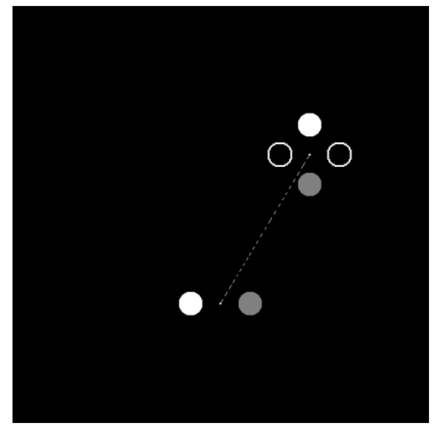
Understanding rigid motion

Rigid motion from (a) to (b):

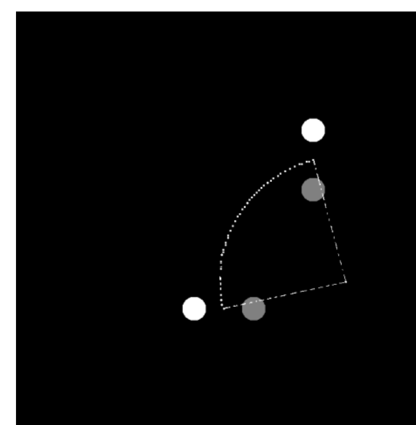


(a) (b)

(I) Rotating, then shifting:



(II) A simple rotating:



Mathematically:

T is a rigid transformation:

$$TX = A(X - C) + D = A(X - S).$$

$$A(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \qquad S = c - A^{-1}d := \begin{pmatrix} s_1 \\ s_2 \end{pmatrix}.$$

C---Rotating center,
Not unique

S---Motion center
Unique, so is the angle

Strategy

Step 1: Find the unique angle.

Angle of motion.

Step 2: Find the unique center for the motion.

Center of motion.

Method

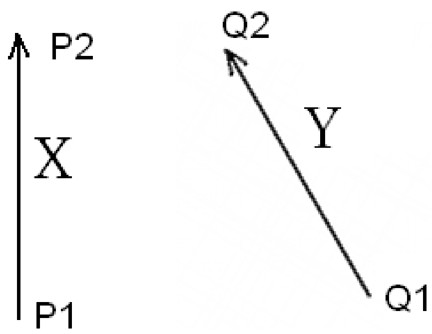
- Find the angle and center for simple image: Two-point rotating model.
- For complicated images: Active contour for catching characteristics (not necessarily the edges).

Two-point rotating model:

Vector rotating

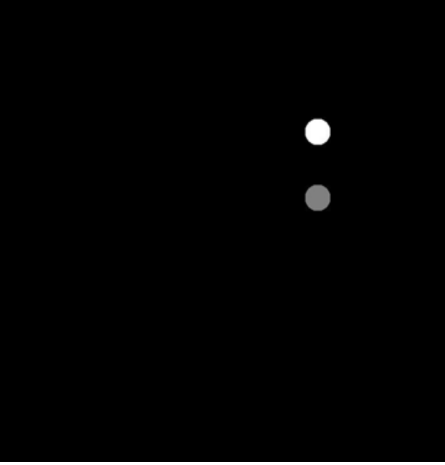
$$\mathbf{X}\cdot\mathbf{Y} = |\mathbf{X}|\cdot|\mathbf{Y}|\cdot\cos(\theta_*)$$

$$\theta_* = \cos^{-1}\frac{\mathbf{X}\cdot\mathbf{Y}}{|\mathbf{X}|\cdot|\mathbf{Y}|}.$$

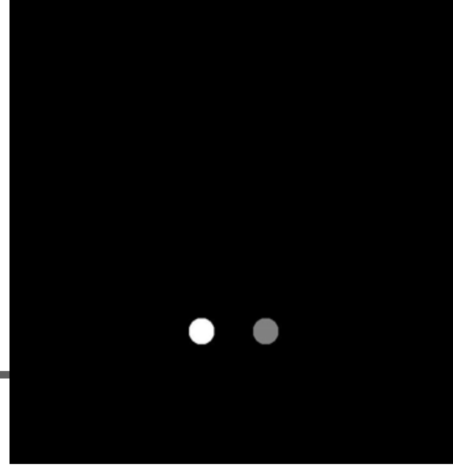


$$A(\theta_*) = \begin{pmatrix} \cos\theta_* & \sin\theta_* \\ -\sin\theta_* & \cos\theta_* \end{pmatrix}.$$

$$\mathbf{S} = (A_* - id)^{-1}\cdot(A_*\mathbf{P}_1 - \mathbf{Q}_1),\ \ or\ \ \mathbf{S} = (A_* - id)^{-1}\cdot(A_*\mathbf{P}_2 - \mathbf{Q}_2).$$

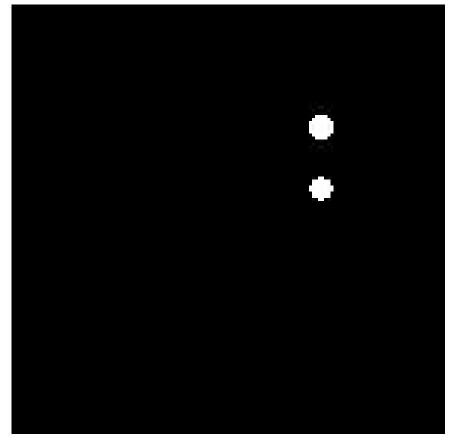


a

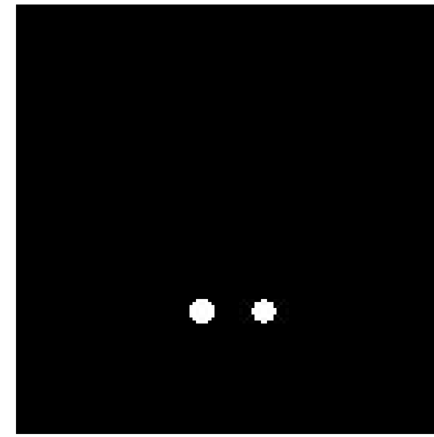


(b)

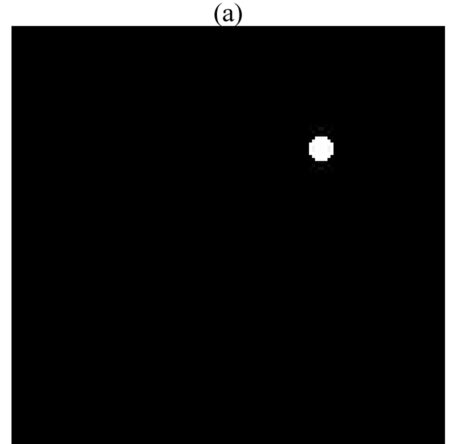
Two characteristics:



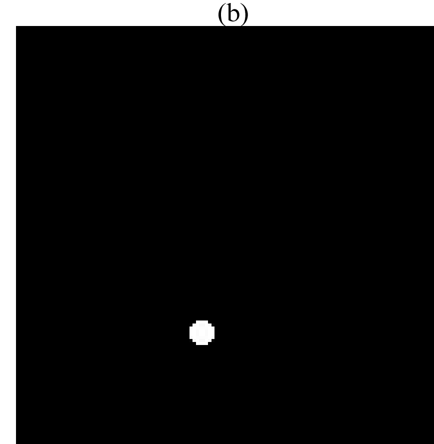
(a)



(b)



(c)



(d)

Centroid:

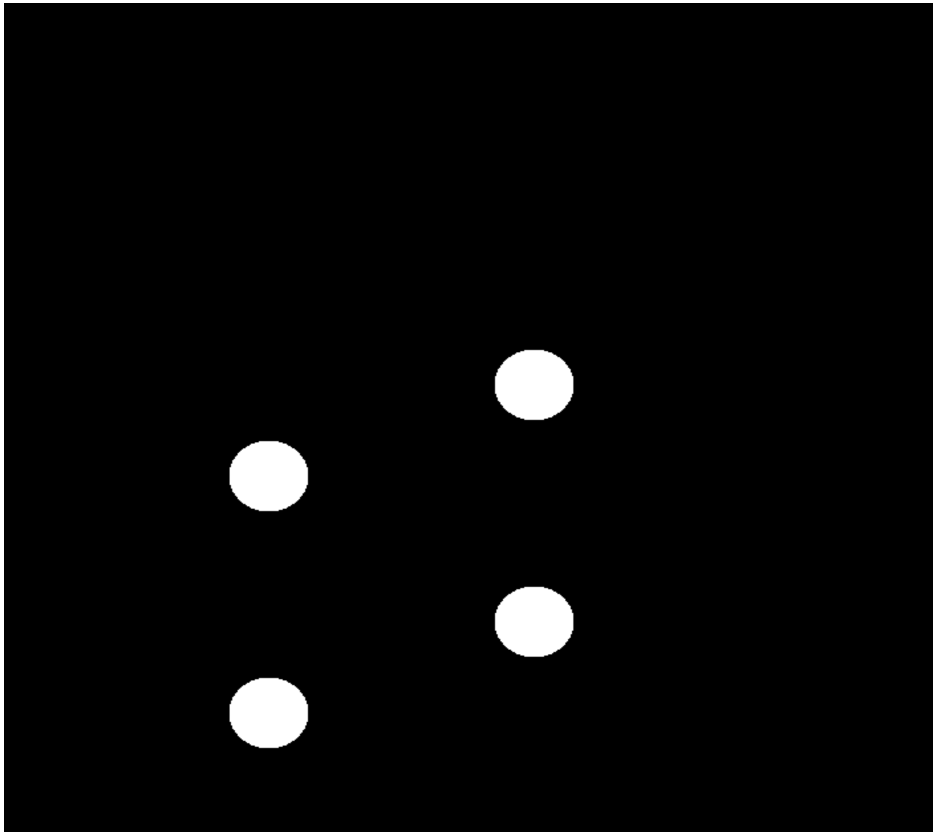
$$x_c = \frac{\int_\Omega x f(x,y)\,dxdy}{\int_\Omega dxdy}, \qquad y_c = \frac{\int_\Omega y f(x,y)\,dxdy}{\int_\Omega dxdy}.$$

First characteristics: (a), (b) yield P1, Q1.

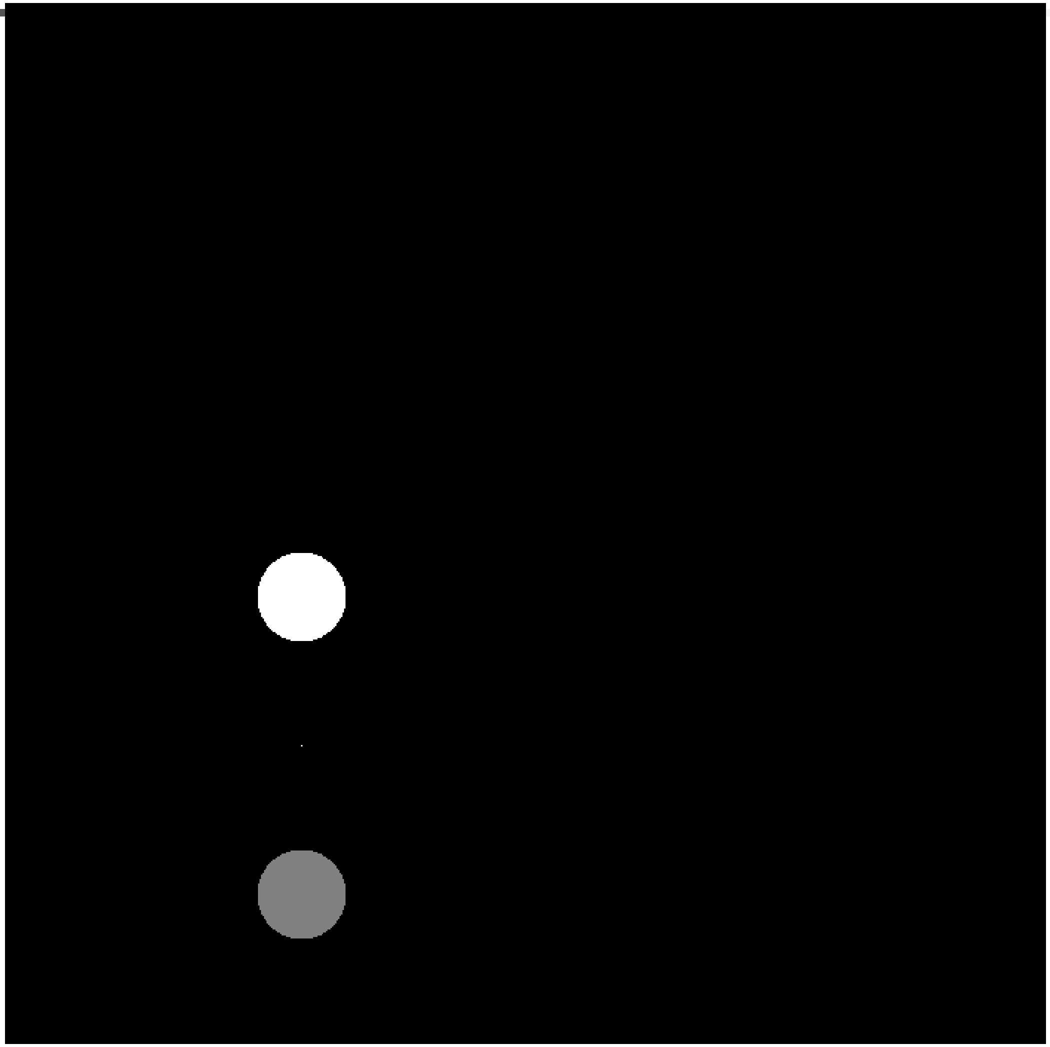
Second characteristics: (c), (d) yield P2, Q2.

Two points is necessary

- Rotating 180º or no rotating at all:

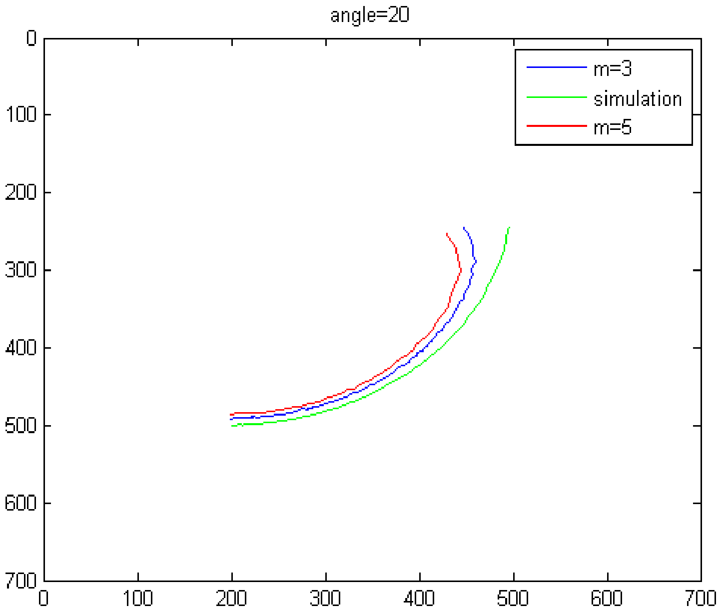


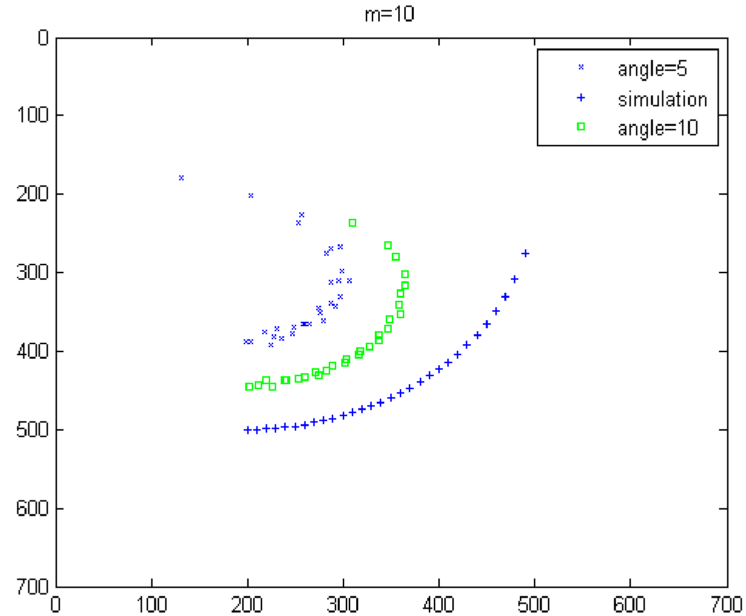
Recall the synthetic motion



Results

Not for tracking!



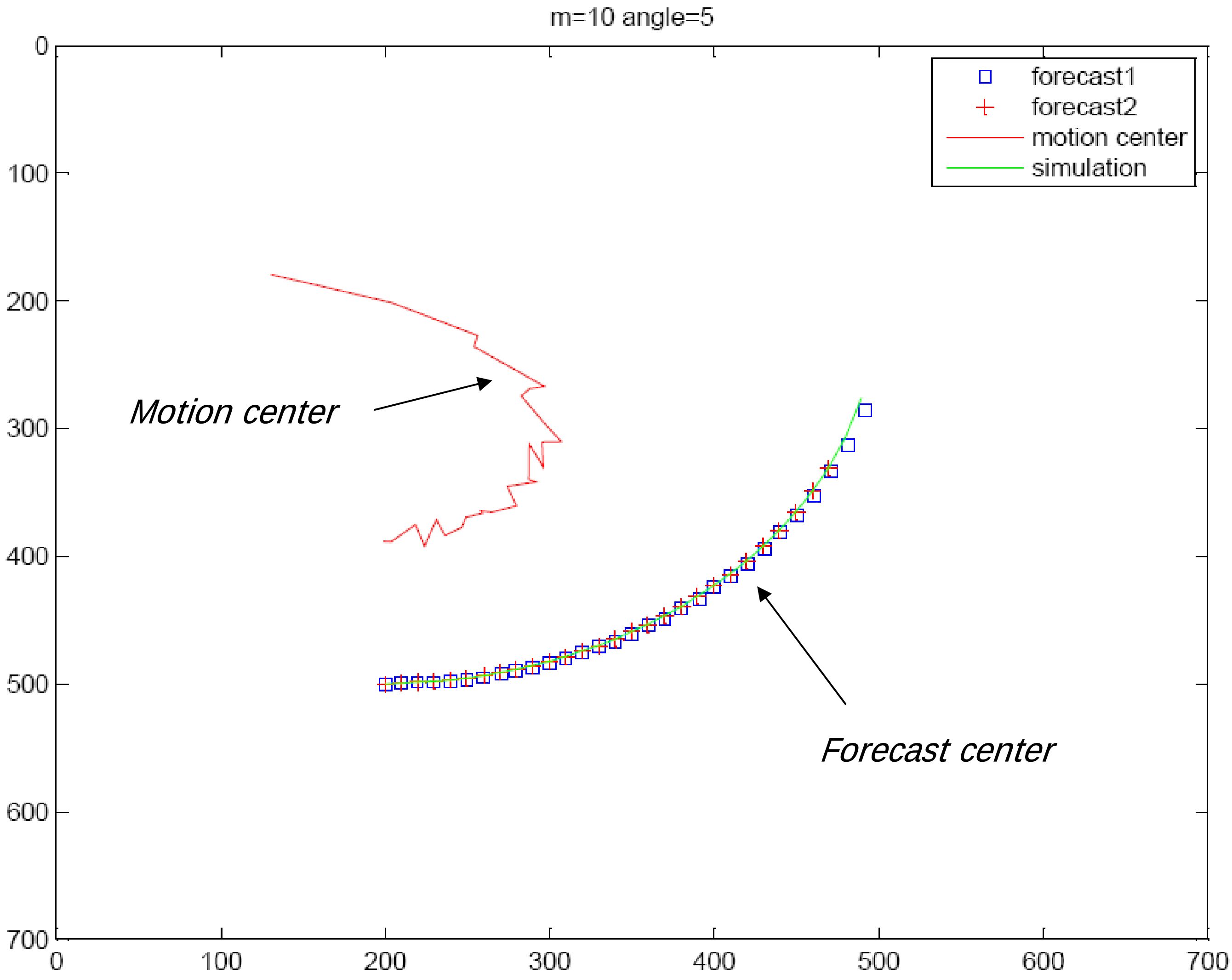


As rotating speed/shifting speed becomes smaller, the difference between the center of motion and the center of rotating gets larger.

Computing motion center and rotating angle

Position	Motion center1	Motion center2	Angle
1	(198.8864,388.3416)	(200.5,388.2728)	4.88
2	(203.4712,388.7125)	(201.5,388.9712)	5.19
3	(218.3488,375.2252)	(215.3025,375.895)	4.65
4	(223.9468,391.9682)	(225.6064,391.4468)	5.43
5	(227.3262,381.7578)	(228.8984,381.1016)	5.00
6	(231.2535,371.1436)	(231.9965,370.7553)	4.87
7	(236.1746,383.6634)	(233.8603,385.1397)	5.01
8	(246.4136,377.4045)	(247.2037,376.7963)	5.10
9	(249.1863,369.2876)	(252.3137,366.415)	4.86
10	(259.1863,366.2876)	(256.3137,369.415)	4.86

Nowcasting



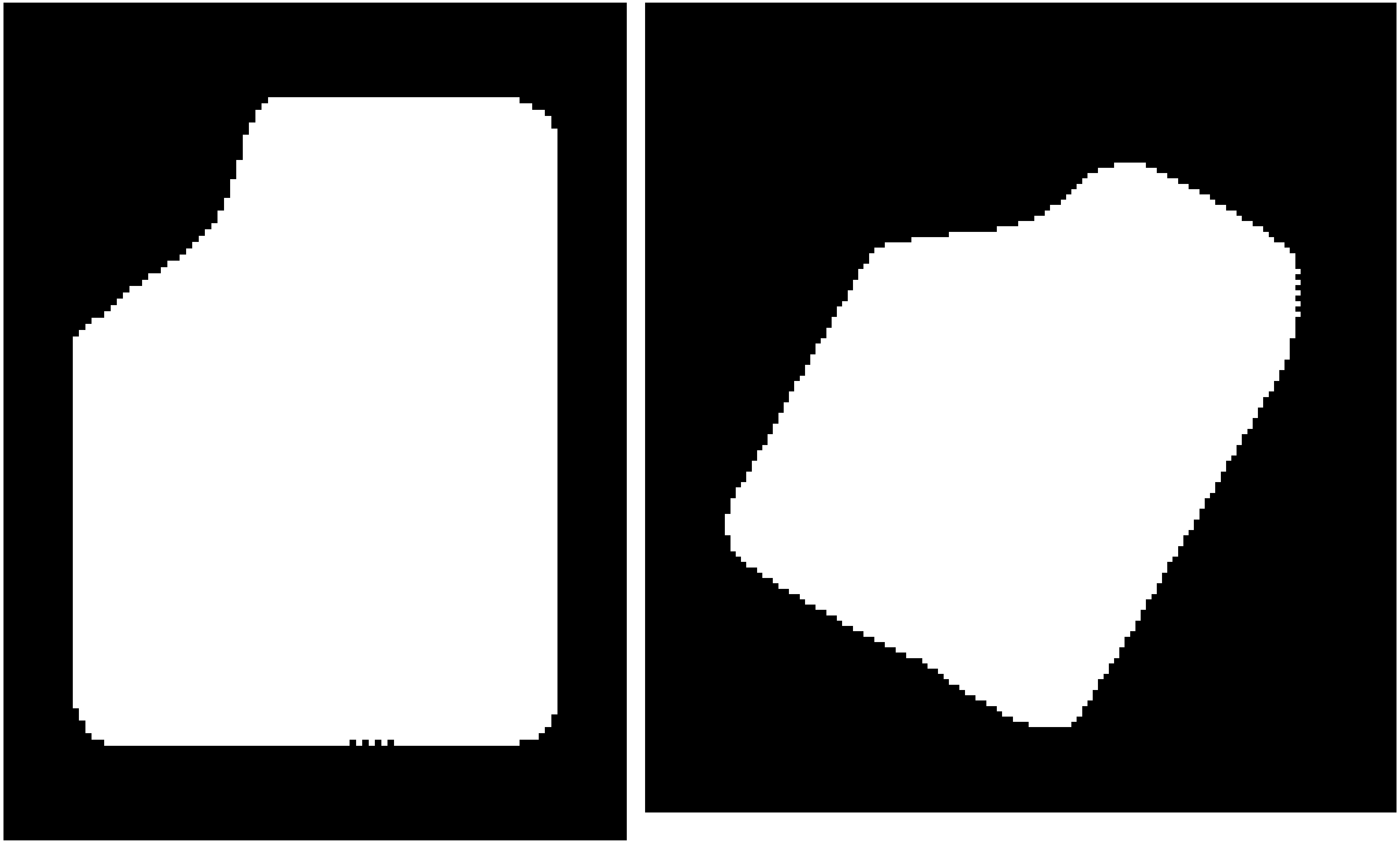
m=10 angle=5
forecast1
forecast2
motion center
simulation
Motion center
Forecast center
0
100
200
300
400
500
600
700

Real images

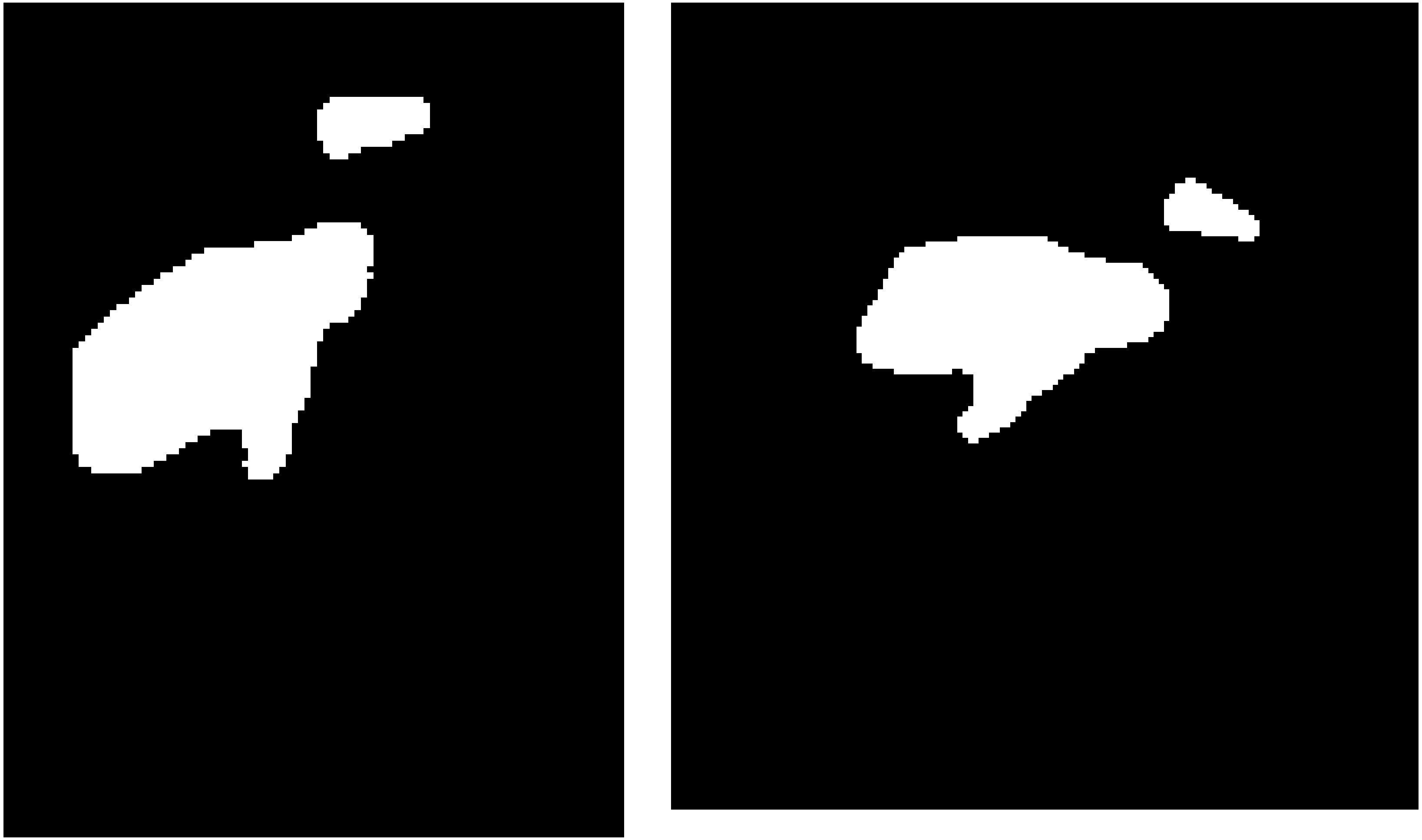
1. Extract intrinsic characteristics of an image

2. Compute the center of mass, then the angle of motion, and the center of motion.

Static images



First characteristics



Second characteristics

How we do that.

Active contour. Similar to Chan-Vese model:

$$I_1(C, c_1, c_2) := \int_{inside(C)} |u_0 - c_1|^2 \, dxdy + \int_{outside(C)} |u_0 - c_2|^2 \, dxdy$$
$$+ \mu \cdot (length(C)) + \nu \cdot (Area(inside(C))).$$

Using level-set method:

$$\begin{cases} \dfrac{\partial \Phi}{\partial t} = \delta(\Phi)\{\mu \cdot div(\dfrac{\nabla \Phi}{|\nabla \Phi|}) - \nu - (u_0 - c_1(\Phi))^2 + (u_0 - c_2(\Phi))^2\} \quad in \quad \Omega \times (0, \infty), \\ \Phi(x, y, 0) = \Phi_0(x, y) \quad in \quad \Omega, \\ \dfrac{\delta(\Phi)}{|\nabla \Phi|} \dfrac{\partial \Phi}{\partial n} = 0 \quad on \quad \partial\Omega \times (0, \infty) \end{cases}$$

Our model

Energy functional: M=max u_0

$$I_{our}(C,c) := \int_{inside(C)} |u_0 - \alpha \cdot M|^2 \, dxdy + \int_{outside(C)} |u_0 - c|^2 \, dxdy$$
$$+ \mu \cdot (length(C)) + \nu \cdot (Area(inside(C))).$$

Level-set approach:

$$J_{our}(\Phi,c) := \int_\Omega |u_0 - \alpha \cdot M|^2 \, H(\Phi(x,y))dxdy + \int_\Omega |u_0 - c|^2 \, (1 - H(\Phi(x,y)))dxdy$$
$$+ \mu \int_\Omega \delta(\Phi(x,y)) \, |\nabla \Phi(x,y)| \, dxdy + \nu \int_\Omega H(\Phi(x,y))dxdy,$$

Flow equation

$$
\begin{cases}
\dfrac{\partial \Phi}{\partial t} = \delta_\varepsilon(\Phi)\{\mu \cdot div(\dfrac{\nabla \Phi}{|\nabla \Phi|}) - \nu \\
\quad -(u_0 - \alpha \cdot M)^2 + (u_0 - c(\Phi))^2\} & in \quad (0,\infty)\times\Omega, \\
\Phi(x,y,0) = \Phi_0(x,y) & in \quad \Omega, \\
\dfrac{\delta_\varepsilon(\Phi)}{|\nabla \Phi|}\dfrac{\partial \Phi}{\partial n} = 0 & on \quad \partial\Omega.
\end{cases}
$$

Catch the first characteristic

$$\alpha = 0.2.$$

Catch the second characteristic

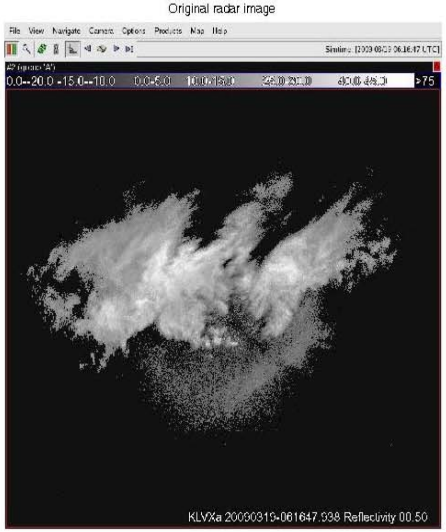
$$\alpha = 1.0.$$

Why new model?

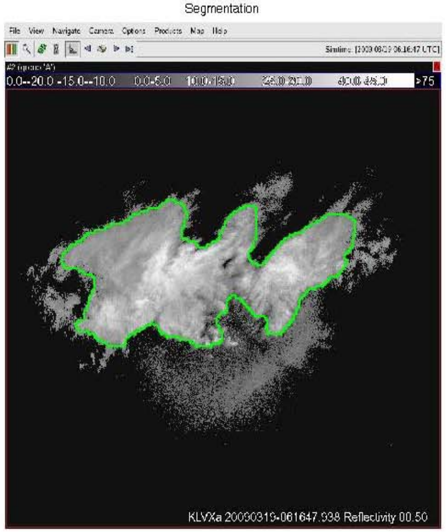
- More efficient to skip radar noise:

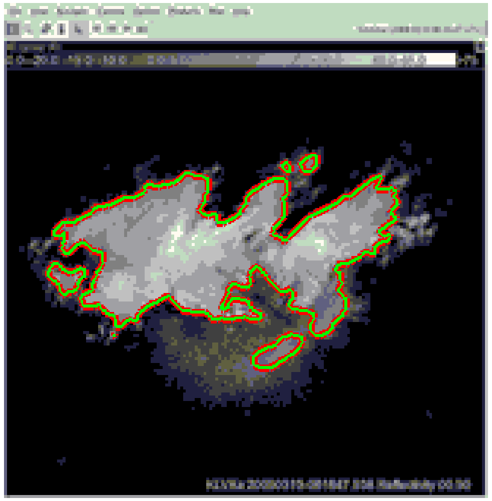
Original image | Our segmentation | CV-segmentation

Original radar image

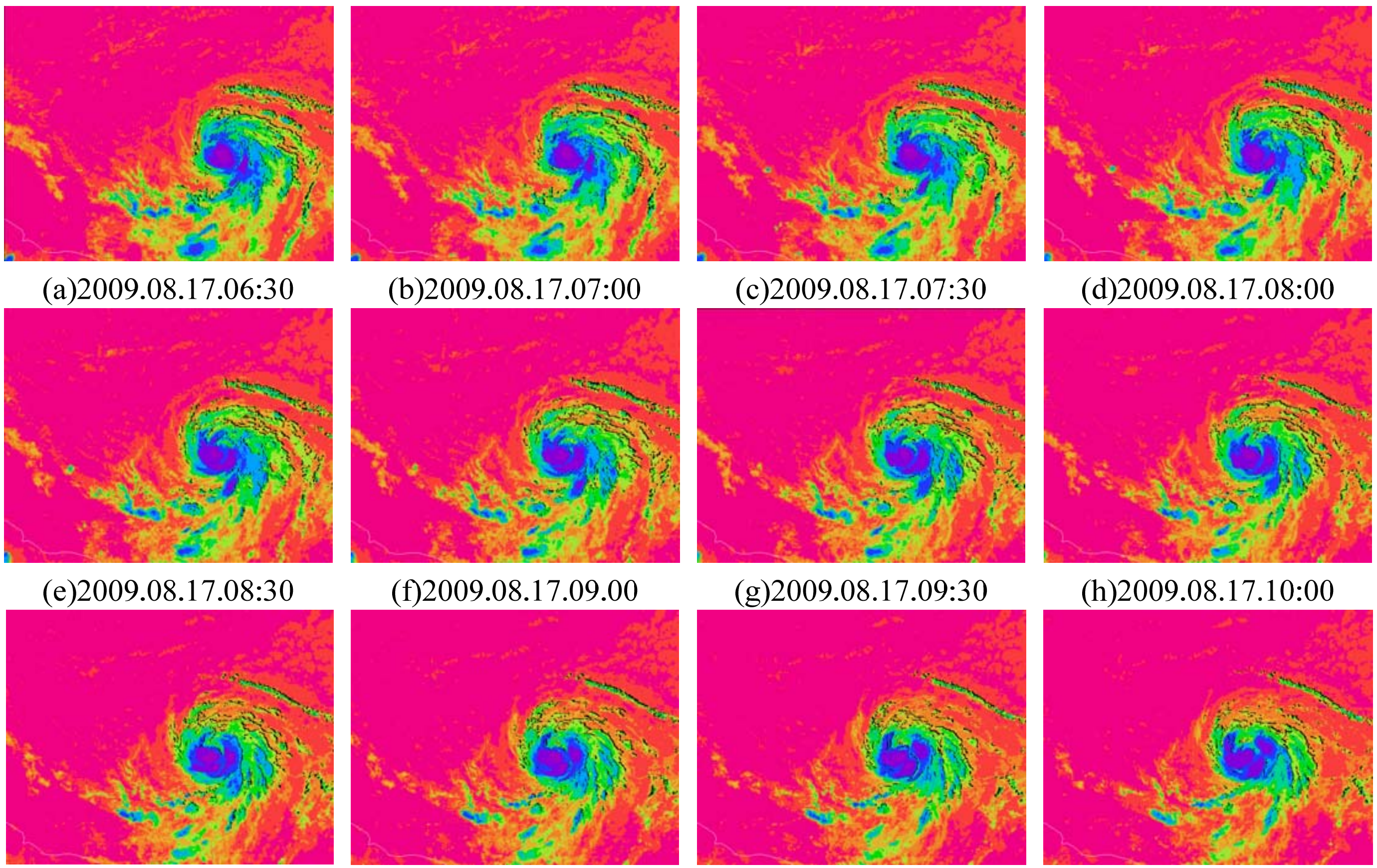


Segmentation





Hurricane Bill approaches Florida coast, August 17-19, 2009



(a)2009.08.17.06:30
(b)2009.08.17.07:00
(c)2009.08.17.07:30
(d)2009.08.17.08:00
(e)2009.08.17.08:30
(f)2009.08.17.09.00
(g)2009.08.17.09:30
(h)2009.08.17.10:00
(i)2009.08.17.10:30
(j)2009.08.17.11:30
(k)2009.08.17.12:00
(l)2009.08.17.12:30

First characteristic images

(a)2009.08.17.06:30

(b)2009.08.17.07:00

(c)2009.08.17.07:30

(d)2009.08.17.08:00

(e)2009.08.17.08:30

(f)2009.08.17.09.00

(g)2009.08.17.09:30

(h)2009.08.17.10:00

(i)2009. . . :

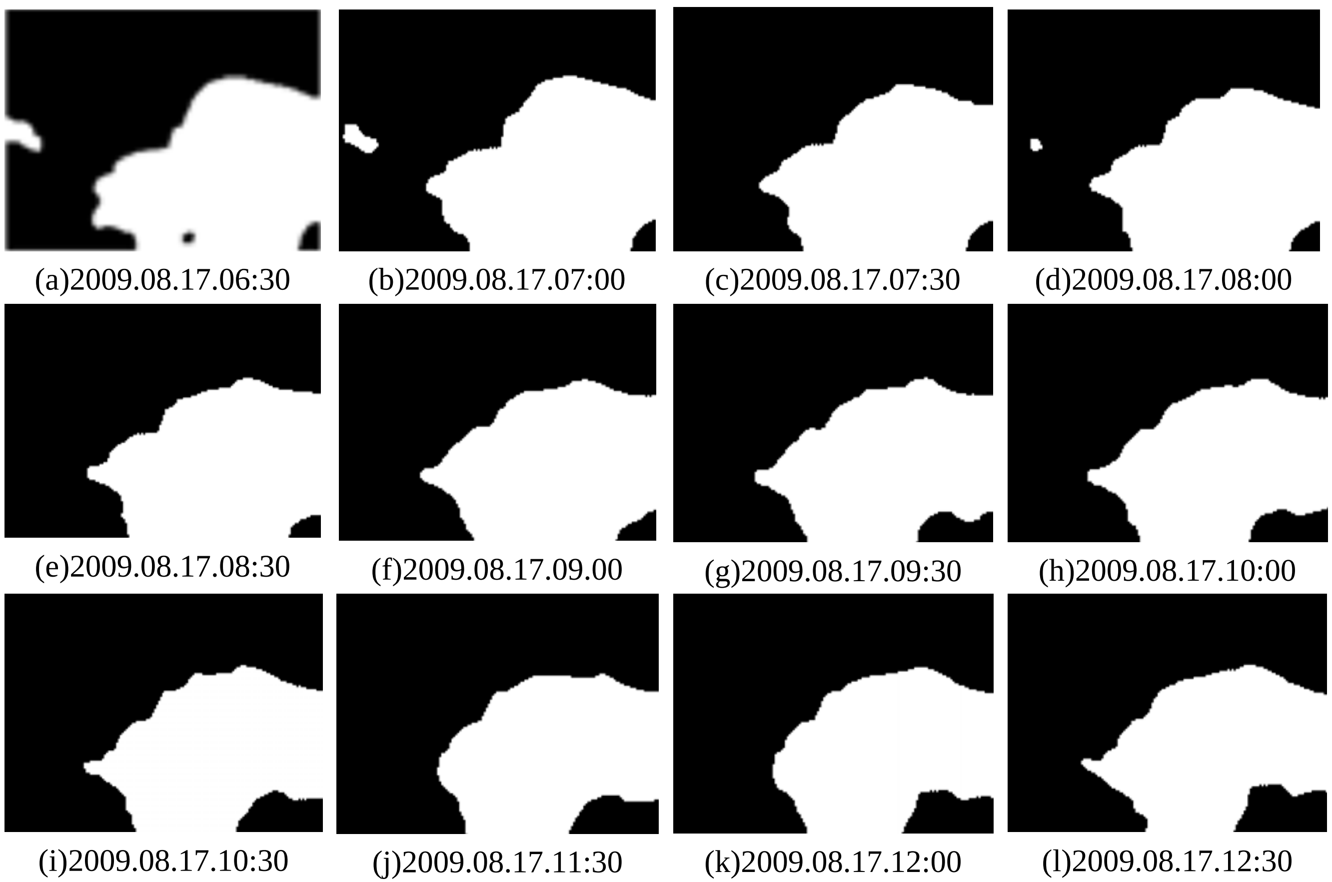
(j)2009.08.17.11:30

(k)2009.08.17.12:00

(l)2009.08.17.12:30

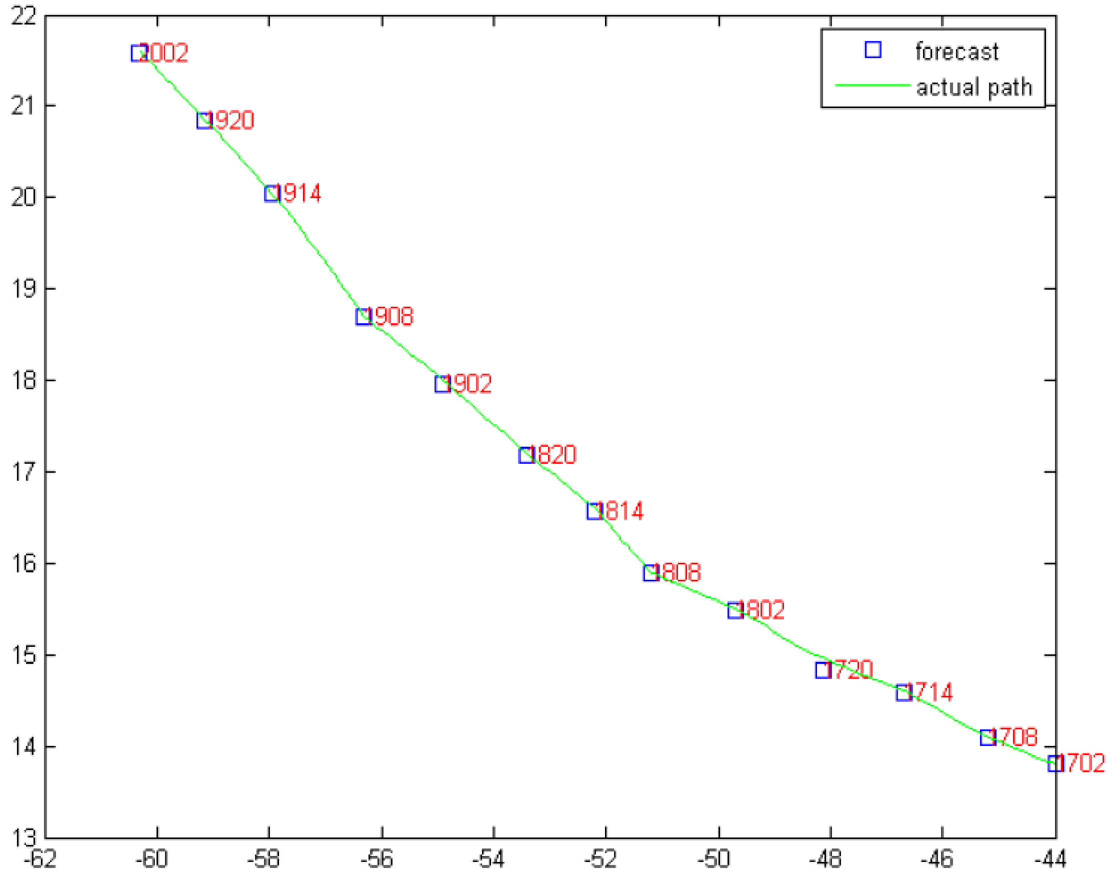
Alpha=0.9

Second characteristics



Alpha=0.7

Hurricane Bill approaches Florida coast, August 17-19, 2009



Computing the motion center and angle

Time	UT1702	UT1708	UT1714	UT1720	UT1802	UT1808	UT1814
Rotating angle	9.087668	9.643614	14.33013	6.851278	11.9749	17.34119	10.70173
Time	UT1814	UT1820	UT1902	UT1908	UT1914	UT1920	UT2002
Rotating angle	10.70173	9.476946	2.7636	19.95497	9.770217	7.0175	5.788656

Strong points and weak points

- First work in this area.
- Works almost perfectly for rigid motions

- Other motion (affine motion)?
- Other ways to catch two characteristics? (More stable, more automatically, faster.)

Mathematical questions:

- Long time existence of the heat equation.
- Stability of the solution w.r.t. parameter and initial images.
- Convergence for numerical computations (CFL type condition).