Math 3333 **Linear Algebra I** Summer 2008

Name:_____

Homework 9 – Linear Transformations – due Friday, July 18th

YOU MUST SHOW ALL OF YOUR WORK!

1 .) (*8 points*) Are the following functions linear transformations? Why or why not?

a.) (*2 points*) $L\colon \mathbb{R}^2 \to \mathbb{R}^3$ given by $L\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x+2y \\ x-y \\ 3x-y \end{bmatrix}$

b.) (*2 points*) $L\colon \mathbb{R}^3 \to \mathbb{R}$ given by $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = x^2+y^2+z^2$.

c.) (*2 points*) $L\colon M_{2,2} \to \mathbb{R}^2$ given by $L\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} a+b \\ c+d \end{bmatrix}$

d.) (*2 points*) $L\colon P_3 \to P_3$ given by $L(a_0+a_1t+a_2t^2+a_3t^3) = 3a_3+a_1t^2+2a_2t^3$

2 .) (*4 points*) Find the standard matrix representing the linear transformation $L\colon \mathbb{R}^3 \to \mathbb{R}^3$ defined by

$$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+4y \\ -z \\ y+z \end{bmatrix}$$

Check that $L(\mathbf{v}) = A\mathbf{v}$ where A is the standard matrix representing L.

3 .) (*2 points*) (**T**rue or **F**alse) (*circle one, no justification required*) For any linear transformation $L\colon \mathbb{R}^n \to \mathbb{R}^m$ there is an $m \times n$ matrix A such that $L(\mathbf{v}) = A\mathbf{v}$.

4 .) (*8 points*) Consider the linear transformation $L\colon M_{2,2} \to \mathbb{R}^2$ defined by

$$L\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} a+b \\ c+d \end{bmatrix}$$

a.) (*4 points*) Find a basis for $\operatorname{Ker} L$. Is L one-to-one?

b.) (*4 points*) Find a basis for $\operatorname{Im} L$. Is L onto?

5 .) (*2 points*) (**T**rue or **F**alse) (*circle one, no justification required*) If $L\colon V \to W$ is a linear transformation and $\dim V > \dim W$, then L is necessarily not one-to-one.