Math 3333 **Linear Algebra I** Summer 2008

Name:____

Homework 10 – Matrix Representations, Determinants – due Tuesday, July 22nd

YOU MUST SHOW ALL OF YOUR WORK!

1 .) (*8 points*) Consider the linear transformation $L\colon \mathbb{R}^2 \to \mathbb{R}^2$ given by:

$$L\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

and fix the bases $S = T = \left\{\begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \end{bmatrix}\right\}$.

a.) (*4 points*) Find the matrix representing L with respect to S & T.

b.) (*2 points*) Find the coordinates of $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$ with respect to S.

c.) (*2 points*) Use your answer in a and b to find $L\left(\begin{bmatrix} 2 \\ 5 \end{bmatrix}\right)$.

2 .) (*6 points*) Consider the linear transformation $L\colon P_3 \to \mathbb{R}$ defined by $L\big(p(t)\big) = \displaystyle\int_1^2 p(t)\,dt$ and fix bases $S = \{1, t, t^2, t^3\}$ of P_3 and $T = \{1\}$ of $\mathbb{R}$.

a.) (*4 points*) Find the matrix representing L with respect to S & T.

b.) (*2 points*) Use your answer in a to find $\displaystyle\int_1^2 1 + 3t + 4t^2 - 2t^3\,dt$.

3 .) (*2 points*) (**T**rue or **F**alse) (*circle one, no justification required*) Suppose $L\colon \mathbb{R}^n \to \mathbb{R}^n$ is the identity linear transformation (i.e. $L(\mathbf{v}) = \mathbf{v}$ for all $\mathbf{v} \in \mathbb{R}^n$), then for two bases S & T for $\mathbb{R}^n$, the matrix representing L with respect to S & T is the transition matrix $P_{T \leftarrow S}$ from the S basis to the T basis.

4 .) (*8 points*) Consider the following matrices:

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 3 & 5 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

a.) (*3 points*) Compute $\det(A)$.

b.) (*3 points*) Compute $\det(B)$.

c.) (*2 points*) Compute $\det(AB)$.

5 .) (*2 points*) (**T**rue or **F**alse) (*circle one, no justification required*) For any two $n \times n$ matrices A and B, $\det(AB) = \det(BA)$.