

Name: _____

Homework 1 – Matrix Algebra – due Friday, June 6th

YOU MUST SHOW ALL OF YOUR WORK!

1.) (12 points) Consider the following matrices:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix}, \quad D = \begin{bmatrix} 3 & -2 \\ 2 & 5 \end{bmatrix}$$

$$E = \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}, \quad F = \begin{bmatrix} -1 & 2 \\ 0 & 4 \\ 3 & 5 \end{bmatrix}$$

If possible, compute the following:

- (2 points) $C + E$
- (2 points) $CF + B$
- (2 points) AB
- (2 points) BA
- (2 points) $D^2 + AF$
- (2 points) $FD + A$

2.) (6 points) Show that $A\mathbf{x}$ can be expressed as a linear combination of the columns of A where:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \quad \& \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

3.) (6 points) Consider the following matrices:

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 0 & -1 & 2 \\ 3 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 0 & 1 \\ 2 & -1 & 0 \\ 2 & 2 & -1 \end{bmatrix}, \quad D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

- (3 points) Show that $DA = 2A$
- (3 points) Show that $2(AB) = A(2B)$

4.) (6 points) Show that the following matrices are inverses of each other:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 2 \\ -1 & 1 & -1 \end{bmatrix}$$