Worksheet 18
An Introduction to Hypothesis Testing

This is a continuation of Discussion Sheet 18. Please complete that now if you haven’t already done so. This corresponds to §9.5 and §9.6 in the textbook Please read those sections. We will treat all hypothesis tests for a mean as using Student’s t-distribution and as with $\sigma$ unknown. Ignore the parts in the textbook about separate forms of hypothesis tests for large samples with $\sigma$ known or unknown.

1. Instead of the $a$ such that $P(t < -a \text{ or } t > a) = P(|t| > a) = .05$ in Question 6 of Discussion Sheet 18, find a $b$ such that $P(t < -b \text{ or } t > b) = P(|t| > b) = .01$ with $\nu = n - 1 \text{ df}$.

2. If you used this region in Question 1 as a rejection region for your hypothesis test of the $H_0$ in Question 1 of Discussion Sheet 18 instead of the rejection region $P(|t| > a)$ from Question 6 in Discussion Sheet 18, what would you conclude and what would be different about the hypothesis test you had performed?

3. Given the data in Question 1 from Discussion Sheet 18, find a 99% confidence interval for $\mu$. How does this compare with your rejection region in Question 6 of Discussion Sheet 18?
A 100(1-\(\alpha\))% Hypothesis Test for \(\mu\)

A (two-sided) hypothesis test for a population mean, \(\mu\), when the populations standard deviation, \(\sigma\), is unknown but the sample size is \(n > 30\) is as follows:

1. \(H_0: \mu = \mu_0\), where \(\mu_0\) is some given constant
2. \(H_a: \mu \neq \mu_0\), the alternative hypothesis
3. A test statistic \(t = \frac{\bar{y} - \mu_0}{s/\sqrt{n}}\), where \(\bar{y}\), \(s\), and \(n\) come from sample data and \(\mu_0\) comes from \(H_0\).
4. A rejection region, \(|t| > t_{\alpha/2}, \nu\), where \(P(t > t_{\alpha/2}, \nu) = \alpha/2\), \(P(t < -t_{\alpha/2}, \nu) = \alpha/2\) and so \(P(t > t_{\alpha/2}, \nu \text{ or } t < -t_{\alpha/2}, \nu) = P(|t| > t_{\alpha/2}, \nu) = \alpha\).
5. A conclusion: "reject \(H_0\)" or "do not reject \(H_0\)."

4. Suppose we have a random sample of data about \(y\) of size \(n = 25\) that had a sample mean of \(\bar{y} = 28\) and a sample standard deviation of \(s = 4\). We want to know whether or not \(\mu = 25\). Perform a hypothesis test at the 95% level to answer this question.


Homework with Solutions to be Posted: 9.16, 9.20