Comments on Discussion Sheet 8 and Worksheet 8 (§4.2-§4.4)
Expected Values for Discrete Random Variables

Discussion Sheet 8
We have now examined random variables, in particular, discrete random variables. We have looked at their probability distributions. We want to look at the idea of expected value and use that to define the mean of a discrete random variable.

Mean or Expected Value of a Discrete Random Variable

The *mean*, \( \mu \), or *expected value*, \( E(y) \), of a discrete random variable \( y \) is

\[
E(y) = \mu = \sum_{y} y \cdot p(y).
\]

1. Find the expected value, \( E(y) \) for the r.v. in #3 in Discussion Sheet 7.

Comment:

\[
E(y) = \sum_{i=0}^{2} y_i p(y_i) = (0)\left(\frac{1}{4}\right) + (1)\left(\frac{2}{4}\right) + (2)\left(\frac{1}{4}\right) = \frac{4}{4} = 1
\]

2. Find the mean, \( \mu \), of the r.v. in #3 in Discussion Sheet 7.

Comment:

\[
\mu = E(y) = 1
\]

Expected Value of a Function of a Random Variable

The *expected value* of \( g(y) \) of a function of a r.v. \( y \), is

\[
E(g(y)) = \sum_{y} g(y) p(y).
\]

3. Find the expected value of \( y^2 \) for the r.v. in #3 in Discussion Sheet 7.

Comment:

\[
E(y^2) = \sum_{i=0}^{2} y_i^2 \cdot p(y_i) = (0^2)\left(\frac{1}{4}\right) + (1^2)\left(\frac{2}{4}\right) + (2^2)\left(\frac{1}{4}\right) = \frac{4}{4} = 1
\]

4. Find the expected value of \( y^2 + 1 \) for the r.v. in #3 in Discussion Sheet 7.

Comment:

\[
E(y^2 + 1) = \sum_{i=0}^{2} (y_i^2 + 1) \cdot p(y_i) = \sum_{i=0}^{2} y_i^2 \cdot p(y_i) + \sum_{i=0}^{2} p(y_i)
\]

\[
= \sum_{i=0}^{2} (y_i^2) \cdot p(y_i) + \sum_{i=0}^{2} p(y_i) = 1 + 1 = 2
\]

The Variance of a Discrete Random Variable

The *variance* of a discrete r.v. \( y \) is \( \sigma^2 = E[(y - \mu)^2] \).
5. Find the variance of the r.v. \( y \) in #3 in Discussion Sheet 7 by using the definition in the box right above this question.

Comment:
\[
\sigma^2 = E \left[ (y - \mu)^2 \right] = \sum_{i=0}^{2} (y_i - 1)^2 p(y_i) \\
= (0 - 1)^2 \left( \frac{1}{4} \right) + (1 - 1)^2 \left( \frac{2}{4} \right) + (2 - 1)^2 \left( \frac{1}{4} \right) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}
\]

Worksheet 8 (§4.2-§4.4)
Expected Values for Discrete Random Variables

We have now examined random variables, in particular, discrete random variables. We have looked at their probability distributions and the idea of expected value. We used that to define the mean and variance of a discrete random variable. Now we look at some further properties of expected value and of the variance and standard deviation of a discrete random variable.

**Properties of the Expected Value of a Discrete Random Variable**

If \( x \) and \( y \) are discrete r.v.’s and \( c \) is any real number constant:

1. \( E(c) = c \)
2. \( E(cy) = cE(y) \)
3. \( E(g_1(y) + g_2(y) + \ldots + g_n(y)) = E(g_1(y)) + E(g_2(y)) + \ldots + E(g_n(y)) \)

1. Use the definition of expected value of a function of a discrete r.v., \( E \left[ g(y) \right] \), to find the value of \( E(y^2 + 2y + 1) \) for the r.v. \( y \) from Discussion Sheets 7 (and 8).

Comment:
\[
E(y^2 + 2y + 1) = \sum_{i=0}^{2} \left( y_i^2 + 2y_i + 1 \right) p(y_i) = \sum_{i=0}^{2} y_i^2 p(y_i) + \sum_{i=0}^{2} 2y_i p(y_i) + \sum_{i=0}^{2} p(y_i) \\
= \sum_{i=0}^{2} y_i^2 p(y_i) + 2 \sum_{i=0}^{2} y_i p(y_i) + \sum_{i=0}^{2} p(y_i) = \frac{4}{4} + 2 \left( \frac{4}{4} \right) + 1 = 4
\]

2. Use the properties of the expected value of discrete r.v.’s to find the value of \( E(y^2 + 2y + 1) \) for the r.v. \( y \) from Discussion Sheets 7 (and 8).

Comment:
\[
E(y^2 + 2y + 1) = E(y^2) + E(2y) + E(1) = E(y^2) + 2E(y) + E(1) \\
= 1 + 2(1) + 1 = 4
\]

3. Which is easier, the method in #1 or the method in #2?

Comment:
The way in #2 is easier.
Alternate Formula for the Variance of a Discrete Random Variable

The variance of a discrete r.v. $y$ is $\sigma^2 \equiv E[(y - \mu)^2] = E(y^2) - \mu^2$.

4. Find the variance of the r.v. $y$ from Discussion Sheets 7 (and 8) using the alternate formula.

Comment:
$$\sigma^2 \equiv E[(y-1)^2] = E(y^2) - 1^2 = 1 - 1 = 0$$

Standard Deviation of a Discrete Random Variable

The standard deviation of a discrete r.v. $y$ is $\sigma \equiv \sqrt{E[(y - \mu)^2]} = \sqrt{E(y^2) - \mu^2}$.

5. Find the standard deviation of the r.v. $y$ from Discussion Sheets 7 (and 8).

Comment:
$$\sigma = \sqrt{\sigma^2} \equiv \sqrt{E[(y-1)^2]} = \sqrt{E(y^2) - 1^2} = \sqrt{0} = 0$$