

# Algebra Qualifier Syllabus

May 2005

## 1 Group Theory

Vinberg, Chapters 1, 4, 10; Hungerford, Chapter I

1. Basic definition and properties
2. Subgroups
3. Cosets, congruences
4. Normal subgroups
5. Quotient groups
6. Homomorphisms
7. Homomorphism and isomorphism theorems
8. Index, finite index
9. Orders of elements and subgroups
10. Cyclic groups
11. Direct and semidirect products
12. Finite groups, Sylow's theorems

## 2 Linear Algebra

Vinberg, Chapters 2, 5, 6

1. Vector space
2. Subspace

3. Linear independence
4. Span
5. Basis
6. Dimension
7. Linear functions
8. Bilinear and quadratic functions
9. Euclidean and Hermitian spaces
10. Linear operator
11. Matrix of a linear operator
12. Eigenvectors and eigenvalues of a linear operator
13. Characteristic polynomial of a linear operator
14. Linear operators and bilinear functions
15. Linear operators on Euclidean and Hermitian spaces
16. Jordan canonical form

### **3 Ring Theory**

Vinberg, Chapters 1, 9; Hungerford, Chapter III

1. Basic definition and properties
2. Integral domain
3. Fields
4. Ideals (left, right, two-sided)
5. Quotient rings
6. Homomorphisms

## 4 Polynomial Algebra

Vinberg, Chapter 3

1. Construction and basic properties
2. Roots of polynomials
3. Euclidean domains
4. Factorization in a Euclidean domain
5. Polynomials with rational coefficients
6. Polynomials of several variables
7. Field of rational fractions

## 5 Commutative Algebra

Vinberg, Chapter 9

1. Abelian groups (finitely generated)
2. Principal ideal domains
3. Modules over principal ideal domains
4. Noetherian rings
5. Prime factorization in Noetherian rings
6. Unique factorization domains
7. Maximal and prime ideals
8. Radical
9. Algebraic elements
10. Minimal polynomial
11. Algebraic field extensions
12. Finite field extensions
13. Splitting field of a polynomial