1. Find the splitting field of $x^3 - 1$ over $\mathbb{Q}$. Express your answer in the form $\mathbb{Q}(a)$.

2. **a)** Describe the elements of $\mathbb{Q}(\pi)$.

   **b)** Let $F = \mathbb{Q}(\pi^3)$. Find a basis of $F(\pi)$ over $F$.

3. Show that $\mathbb{Q}(\sqrt{2})$ is not ring isomorphic to $\mathbb{Q}(\sqrt{3})$.

4. Find all the ring automorphisms of $\mathbb{Q}(\sqrt[3]{5})$.

5. **a)** Find the degree and a basis for $\mathbb{Q}(\sqrt{3}+\sqrt{5})$ over $\mathbb{Q}(\sqrt{15})$.

   **b)** Find the degree of $\mathbb{Q}(\sqrt{3}+\sqrt{5})$ over $\mathbb{Q}$.

6. If $v$ is algebraic over $K(u)$ for some $u \in F$ and $v$ is transcendental over $K$, then $u$ is algebraic over $K(v)$.

7. If $u \in F$ is algebraic of odd degree over $K$, then so is $u^2$ and $K(u) = K(u^2)$.