1. Let A_1 and A_2 be rings, and $A = A_1 \oplus A_2$ is their direct sum. Show that every ideal I of A has the form $I = \{(a,b) | a \in I_1, b \in I_2\}$, where I_1 is an ideal in A_1 and I_2 is an ideal in A_2.

2. Let A be a PID, M left A-module, $p \in A$ a prime element. Let $pM = \{pm | m \in M\}$, and $Ann(p) = \{m \in M | pm = 0\}$. Show

   a) $A/(p)$ is a field

   b) pM, $Ann(p)$ are submodules of M

   c) M/pM is a vector space over $A/(p)$ with

   $$(a + (p))(m + pM) = am + pM$$

   d) $Ann(p)$ is a vector space over $A/(p)$ with

   $$(a + (p))m = am$$

3. Let A be a PID. Show that

   a) every proper ideal is a product $P_1 P_2 ... P_n$ of maximal ideals;

   b) if $P_i = (p_i)^{n_i}$, where p_i is a prime element in A, then $P_1 P_2 ... P_n = P_1 \cap P_2 \cap ... \cap P_n$.

4. **a)** Show that the set of all nilpotent elements of a commutative, associative ring A with 1 is an ideal.

   b) Show that $A/radA$ has no nilpotent elements.

5. Show that $< x > \subset \mathbb{Z}[x]$ is a prime ideal, but not a maximal ideal.