Homework 3

MATH 5353

October 25, 2004

1. Let G be a group such that the intersection of all its subgroups which are different from e is a subgroup different from e. Prove that every element in G has a finite order.

2. - If H is a subgroup of G and $a \in G$, let $aHa^{-1} = \{aha^{-1} | h \in H\}$. Show aHa^{-1} is a subgroup in G.
   - If H is a subgroup of finite index in G, prove that there is only a finite number of distinct subgroups of G of the form aHa^{-1}.

3. If H is a subgroup of a group G of index 2 prove that H is a normal subgroup of G.

4. Suppose N and M are two normal subgroups of G and that $N \cap M = \{e\}$. Show that for any $n \in N, m \in M$ we have $nm = mn$.

5. If H is a cyclic subgroup of a group G and H is normal in G, then every subgroup of H is normal in G.

6. Prove by example, that we can find three groups $E \subset F \subset G$, where E is normal in F, F is normal in G, but E is not normal in G.

7. If N is normal in G and $a \in G$ is of order $o(a)$, prove that the order m of aN in G/N is a divisor of $o(a)$.