Math 3333, Final

December 12, 2007

Show all your work to receive full credit. The use of books and notes is not allowed. Good luck!

I. (15 pts) Show that the matrix $\begin{bmatrix} 6 & -4 \\ 8 & -6 \end{bmatrix}$ is diagonalizable and find a diagonal matrix similar to it.

II. (15 pts) Given the matrix $A = \begin{bmatrix} -1 & -2 \\ 4 & 5 \end{bmatrix}$

a) Find the characteristic polynomial $p(\lambda)$ of A.

b) Find the eigenvalues of A.

c) For each eigenvalue from part b) find an associated eigenvector.

III. (10 pts) Let $L : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear operator for which

$$L(\begin{bmatrix} 1 \\ 1 \end{bmatrix}) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}, L(\begin{bmatrix} -1 \\ 1 \end{bmatrix}) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}.$$

Find $L(\begin{bmatrix} -1 \\ 5 \end{bmatrix})$.

IV. (15 pts) Let $L : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation defined by

$$L(\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}) = \begin{bmatrix} u_1 + u_2 \\ u_2 - u_3 \end{bmatrix}.$$

a) Find a basis for the kernel of L.

b) Find a basis for the range of L.

V. (15 pts) Let $L : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by

$$L(\begin{bmatrix} x \\ y \end{bmatrix}) = \begin{bmatrix} x \\ -y \end{bmatrix}.$$

a) Find the matrix representation A of L with respect to the standard basis $S = \{\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}\}$.

b) Find the matrix representation B of L with respect to the ordered basis $T = \{\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix}\}$.

VI. (15 pts) Given the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & 3 \\ 0 & 4 & -1 \end{bmatrix}$

a) Find a basis for the row space of A.

b) What is the rank of A?

c) Find a basis for the null space of A.