Math 3333, Test II

November 5, 2008

Show all your work to receive full credit. The use of books and notes is not allowed. Good luck!

I. (20 pts) Which of the following subsets of the vector space $\mathbb{R}^3$ are subspaces? Explain your answers.

a) The set of vectors $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$, such that $a = 2b$.

b) The set of vectors $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$, such that $a + b = 2$.

II. (20 pts) Is the set $S = \{ \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \}$ a basis of $\mathbb{R}^3$? Prove your answer.

III. (20 pts) Find a basis for and the dimension of the solution space of the homogeneous system

$$\begin{array}{ccccccccccc} x_1 & + & 2x_2 & + & x_3 & + & 2x_4 & + & x_5 & = & 0 \\ x_1 & + & x_2 & + & 2x_3 & + & x_4 & + & x_5 & = & 0 \end{array}$$

IV. (20 pts)
Let $S = \{\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $T = \{\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ be ordered bases of $\mathbb{R}^2$. Let $v = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$.

a) Find the coordinate vector of v with respect to the basis T.

b) Find the transition matrix $P_{S \leftarrow T}$.

c) Find the coordinate vector of v with respect to the basis S using $P_{S \leftarrow T}$¿

V. (20 pts) Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & -1 & 0 & 0 \\ 0 & 1 & -1 & 2 \end{bmatrix}$.

a) Find the rank of A.

b) Find the dimension of the null space of A.