Math 3333, Test II

October 31, 2007

Show all your work to receive full credit. The use of books and notes is not allowed. Good luck!

I. (20 pts) Which of the following subsets of the vector space P_2 are subspaces? Explain your answers.

The set of polynomials of the form

a) $a_2t^2 + a_1t + a_0$, where $a_0 = 0$

b) $a_2t^2 + a_1t + a_0$, where $a_0 = 2$

c) $a_2t^2 + a_1t + a_0$, where $a_0 = a_2 + a_1$

II. (20 pts) Find all real values c, such that the vectors $v_1 = \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 2 \\ 0 \\ c^2+1 \end{bmatrix}$ are linearly dependent.

III. (20 pts) Is the set $S = \{ \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \}$ a basis of $\mathbb{R}^3$? Prove your answer.

IV. (20 pts) Find a basis for and the dimension of the solution space of the homogeneous system

$$\begin{array}{rcrcrcrcl} x_1 & & & + & 2x_3 & + & x_4 & = & 0 \\ x_1 & + & x_2 & + & 2x_3 & + & 2x_4 & = & 0 \\ x_1 & - & x_2 & + & 2x_3 & & & = & 0 \end{array}$$

V. (20 pts) Let $A = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 2 & 4 & 1 & 7 \\ 0 & 0 & 2 & 2 \end{bmatrix}$.

a) Find the rank of A.

b) Find the dimension of the null space of A.