

# Review Problems for Test 1

MATH 2443-006, Spring 04

- Let  $f(x, y) = \sqrt{36 - 4x^2 - 9y^2}$ 
  - Describe and sketch the domain of  $f$ .
  - Since  $P(-1/2, 1)$  is in the domain, there is a level curve for  $f$  at  $C$  which passes through  $P$ . Find the value  $C$ .
- Find the limit or show that it does not exist
  - $\lim_{(x,y) \rightarrow (0,0)} \frac{(x-y)^2}{x^2+y^2}$
  - $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3+y^3}{x^2+y^2}$
- Use linear approximation to approximate a suitable function  $f(x, y)$  and thereby estimate the following
$$(0.99e^{0.02})^8$$
- Consider the surface  $xyz = 30$ 
  - Find the plane tangent to the surface at the point  $(2, 3, 5)$
  - Give a parametric equation for the line normal to the surface at  $(2, 3, 5)$
- Given  $z = e^r \cos \theta$ ,  $r = st$ ,  $\theta = \sqrt{s^2 + t^2}$ , find  $\frac{\partial z}{\partial s}$  and  $\frac{\partial z}{\partial t}$ .
- Find the maximum rate of change of  $f(x, y) = x^2y + y^2z$  at the point  $P(1, 2, -1)$  and the direction in which it occurs.
- Compute the derivative of  $f(x, y, z) = e^x + yz$  at  $P(1, 1, 1)$  in the direction of  $\mathbf{v} = (1, -1, 1)$ .
- Find the local maximum and minimum values and saddle points of the function  $f(x, y) = 4xy + 2x^2y - xy^2$ .

9. Find the absolute maximum and minimum of  $f(x, y) = y\sqrt{x} - y^2 - x + 6y$  on the rectangle  $0 \leq x \leq 9$ ,  $0 \leq y \leq 5$ .
10. Find 3 positive numbers whose sum is 100 and whose product is a maximum.
11. Use Lagrange multipliers to find the maximum and minimum values of the function  $f(x, y) = x^2 + y^2$  subject to the constraint  $x^4 + y^4 = 1$ .