Review problems for Test I

MATH 2433-005, Spring 2005

1. Find an equation of the tangent to the curve $x = 2\sin 2t$, $y = 2\sin t$ at the point $(\sqrt{3}, 1)$.

2. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for the curve $x = t + \ln t$, $y = t - \ln t$. For which values of t is the curve concave upward?

3. Find the points on the curve $x = 10 - t^2$, $y = t^3 - 12t$ where the tangent is horizontal or vertical.

4. Find the length of the curve

   a) $x = \frac{t}{1+t}$, $y = \ln(1+t)$, $0 \le t \le 2$

   b) $x = e^t + e^{-t}$, $y = 5 - 2t$, $0 \le t \le 3$

5. Identify the curve $r = 2\sin\theta + 2\cos\theta$ by finding the Cartesian equation for it.

6. Find polar coordinates of the points given with Cartesian coordinates $(-1, -\sqrt{3})$, $(-2, 3)$.

7. Find the slope of the tangent line to the polar curve $r = \ln\theta$ at the point $\theta = e$.

8. Find the area of the region enclosed by one loop of $r = 4\sin 2\theta$.

9. Determine whether the series is convergent or divergent. If convergent, find its sum.

   a) $\sum_{n=1}^{\infty} \frac{(-4)^{n-1}}{7^n}$

   b) $\sum_{n=1}^{\infty} \frac{(-6)^{n-1}}{5^{n-1}}$

   c) $\sum_{n=1}^{\infty} \ln\left(\frac{5n^2}{n^2+n}\right)$

   d) $\sum_{n=1}^{\infty} \frac{4^n - 3^n}{9^n}$

10. Determine whether the series is convergent or divergent

   a) $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

   b) $\sum_{n=1}^{\infty} \frac{1+\sin n}{10^n}$

   c) $\sum_{n=2}^{\infty} \frac{\sqrt{n}}{n-1}$

   d) $\sum_{n=1}^{\infty} \frac{2n^2+7n}{3^n(n^2+5n-1)}$