Review Problems for the Final

MATH 2433, Spring 2005

1. Test the series for convergence or divergence

   a) $\sum_{n=0}^{\infty} \frac{3^n+4^n}{5^n}$

   b) $\sum_{n=1}^{\infty} \frac{1}{n^{5/2}}$

   c) $\sum_{n=0}^{\infty} \frac{\cos(3n)}{1+(1.5)^n}$

   d) $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{n+1}$

2. Find the radius of convergence and the interval of convergence of the power series

   a) $\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{n^2 5^n}$

   b) $\sum_{n=0}^{\infty} \frac{2^n (x-3)^n}{\sqrt{n+3}}$

3. Find the Taylor series for $f(x) = x^{-2}$ at $a = 1$.

4. Determine whether the given vectors are orthogonal, parallel or neither

   a) $\mathbf{u} =< -3, 9, 6 >$, $\mathbf{v} =< 4, -12, -8 >$

   b) $\mathbf{u} =< 1, -1, 2 >$, $\mathbf{v} =< 2, 2, 0 >$

   c) $\mathbf{u} =< 2, 6, -4 >$, $\mathbf{v} =< 1, -1, 1 >$

5. Find the parametric equations of the line through the points $P(-3, 2, 0)$ and $Q(5, 1, 4)$

6. Find the equation of a plane through the point $(-2, 8, 10)$ and perpendicular to the line $x = 1 + t$, $y = 2t$, $z = 4 - 3t$.

7. Find the equation of a plane that passes through the points $(2, 0, 0)$, $(0, 7, 0)$ and $(0, 0, 4)$.

8. Find at which point the line $x = 1 + 2t$, $y = 4t$, $z = 2 - 3t$ intersects the plane $x - 2y + z = 12$.

9. Change from rectangular coordinates to spherical coordinates $A(-1, 1, \sqrt{6})$, $B(1, \sqrt{3}, 2\sqrt{3})$.

10. Change from spherical coordinates to cylindrical coordinates $A(2\sqrt{2}, 3\pi/2, \pi/2)$, $B(4, \pi/4, \pi/3)$.

11. Write the equation of the surface $y^2 + z^2 = 1$ a) in cylindrical coordinates, b) in spherical coordinates.

12. Evaluate the integral

$$\int_1^4 \sqrt{t}\mathbf{i} + \ln t\mathbf{j} + 1/t^2\mathbf{k}dt$$

13. Find the unit tangent vector $\mathbf{T}(t)$, the unit normal vector $\mathbf{N}(t)$ and the curvature k of the curve

    a) $\mathbf{r}(t) =< \sqrt{2}t, e^t, e^{-t} >$

    b) $\mathbf{r}(t) =< t^2, 2t, \ln t >$

14. Find the velocity and position vectors of a particle that has acceleration $\mathbf{a}(t) = -10\mathbf{k}$, initial velocity $\mathbf{v}(0) = \mathbf{i} + \mathbf{j} - \mathbf{k}$ and initial position $\mathbf{r}(0) = 2\mathbf{i} + 5\mathbf{j} - \mathbf{k}$.