Review problems for Test II

MATH 2433-005, Spring 2005

1. Find the radius and the interval of convergence of the power series

   a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}(x+4)^n$

   b) $\sum_{n=1}^{\infty} \frac{6^{-n}}{n}x^n$

2. Express the function as a power series

   a) $\frac{1}{1-2x}$

   b) $\frac{1}{(1-2x)^2}$

   c) $\frac{x^2}{(1-2x)^2}$

3. Find the Taylor series for $f(x) = \sin x$ at $a = \pi/2$. What is the radius of convergence of this series?

4. **a)** Approximate $f(x) = \ln(1+2x)$ by 3^{rd} degree Taylor polynomial T_3 at $a = 1$.

   b) Use Taylor's Inequality to estimate the accuracy of the approximation on the interval $0.5 \leq x \leq 1.5$.

5. For a certain power series $\sum c_n x^n$, it is known that $\sum c_n$ is convergent.

   a) If $\sum(-1)^n c_n$ is divergent, what can be said about the radius of convergence of this series?

   b) If $\sum(-2)^n c_n$ is divergent, what can be said about the radius of convergence?

6. If $\mathbf{a} =< -3, -4, -1 >$ and $\mathbf{b} =< 6, 2, -3 >$, find $|\mathbf{a}|$, $\mathbf{a}+\mathbf{b}$, $3\mathbf{a}+4\mathbf{b}$, $\mathbf{a}\cdot\mathbf{b}$, $\cos\theta$, and $\mathbf{a}\times\mathbf{b}$.

7. Find the unit vector in the direction of $\mathbf{a} = 8\mathbf{i} - \mathbf{j} + 2\mathbf{k}$.

8. Determine whether the given vectors are orthogonal, parallel or neither

a) $\mathbf{a} =< 4, 6 >$, $\mathbf{b} =< -3, 2 >$

b) $\mathbf{a} = -\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$, $\mathbf{b} = 2\mathbf{i} - 4\mathbf{j} - 8\mathbf{k}$

9. Find a unit vector orthogonal to $\mathbf{i} + 2\mathbf{j}$ and $\mathbf{j} + 2\mathbf{k}$.

10. Find the area of a parallelogram with vertices $A(0, 1, 2)$, $B(0, 2, 5)$, $C(2, 7, 5)$ and $D(2, 6, 2)$.