Review for Midterm I

MATH 2433-003, Honors

1. A cycloid is given by parametric equations $x = r(\theta - \sin\theta)$, $y = r(1 - \cos\theta)$ (r is a fixed number)

   a) Find the tangent to the curve at $\theta = \pi/3$.

   b) Find the length of one arc of the cycloid.

   c) Find the area of the region bounded by one arc of the cycloid and the x-axis.

2. Show that $x = \cos t$, $y = \sin t \cos t$ has two tangents at $(0, 0)$ and find their equations.

3. The curve is given by a polar equation $r = \sin 2\theta$.

   a) Sketch the curve.

   b) Find the slope of the tangent to this curve at $\theta = \pi/4$.

   c) Find the area of the region enclosed by one loop of the curve.

   d) Find the length of one loop.

4. Find the foci and vertices and sketch the graph

   a) $4x^2 - y^2 = 16$

   b) $6y^2 + x - 36y + 55 = 0$

5. Find an equation of the ellipse with foci $(3, \pm 2)$ and major axis with length 8.

6. Determine if the sequence converges or diverges. If converges, find the limit.

   a) $a_n = \frac{\sqrt{n}}{1+\sqrt{n}}$

   b) $a_n = \sin(n\pi/2)$

   c) $a_n = \ln(n + 1) - \ln n$

7. Show that the sequence defined by $a_1 = 1$, $a_{n+1} = 3 - \frac{1}{a_n}$ is increasing and bounded above by 3. Deduce that $\{a_n\}$ is convergent and find its limit.

8. Determine if the series is convergent or divergent.

   a) $\sum_{n=1}^{\infty} \frac{(-6)^{n-1}}{37^n}$

   b) $\sum_{n=1}^{\infty} \frac{arctann}{1+n^2}$

   c) $\sum_{n=1}^{\infty} \frac{1}{n \ln^2 n}$

   d) $\sum_{n=1}^{\infty} \frac{1}{n^3+n+11}$

   e) $\sum_{n=1}^{\infty} \frac{n+5}{\sqrt{n^5+n^2}}$

9. Show that if $a_n > 0$ and $\lim_{n\to\infty} na_n \neq 0$, then $\sum a_n$ is divergent.