Review for the Final

MATH 2433-003, Honors

1. Find the radius and the interval of convergence of

$$\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n3^n}$$

2. Show that if the limit $\lim_{n\to\infty} \sqrt{|c_n|} = c$, then the radius of convergence of the power series $\sum c_n x^n$ is $R = \frac{1}{c}$.

3. Use series to approximate the definite integral

$$\int_0^1 \sin(x^2)dx$$

within 3 decimal places.

4. Find a vector orthogonal to the plane through the points P, Q, R and the area of triangle PQR, if $P(1, 0, -1)$, $Q(2, 4, 5)$ and $R(3, 1, 7)$.

5. Find the parametric equations for the line of intersection of the planes $z = x + y$ and $x - 3y + z = 2$.

6. Find an equation of a plane that passes through the point $(6, 0, -2)$ and contains the line $x = 4 - 2t$, $y = 3 + 5t$, $z = 7 + 4t$.

7. Classify the surface and sketch it

   a) $9x^2 + y^2 - z^2 - 2y + 2z = 0$

   b) $z = 4x^2 + 4y^2 + 1$

8. Change from cylindrical to spherical coordinates

   a) $(2, 0, 0)$

   b) $(8, \pi/6, \pi/2)$

9. Identify the surface

   a) $\rho \cos \phi = 2$

   b) $r^2 + z^2 = 25$

10. Find the length of the curve $\mathbf{r}(t) = (t^2, \sin t - t\cos t, \cos t + t \sin t)$, $0 \leq t \leq \pi$.

11. Find the curvature of $\mathbf{r}(t) = (\sqrt{2}t, e^t, e^{-t})$ at the point $(0, 1, 1)$.

12. What force is required so that a particle of mass m has the position function $\mathbf{r}(t) = t^3\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$?

13. A ball is thrown at an angle $45°$ to the ground. If the ball lands $90m$ away, what was the initial speed of the ball?

14. Find the velocity, acceleration and speed of a particle with the position function

$$\mathbf{r}(t) = e^t \cos t\mathbf{i} + e^t \sin t\mathbf{j} + te^t\mathbf{k}?$$