Review problems, MATH 2423

Test I

September 22, 2005

1. Estimate the area under the graph of $f(x) = 25 - x^2$ from $x = 0$ to $x = 5$ using five approximating rectangles and right endpoints. Sketch the graph and the rectangles.

2. If $f(x) = 3x - 7$, $0 \leq x \leq 3$, evaluate the Riemann sum with $n = 6$, taking the sample points to be left end points.

3. Use properties of integrals to estimate

$$\int_0^2 \sqrt{x^3 + 1} dx$$

4. Compute the integrals

   a) $\int_1^9 \frac{3x-2}{\sqrt{x}} dx$

   b) $\int_0^{3\pi/2} |\sin x| dx$

   c) $\int_0^2 y^2 \sqrt{1 + y^3} dy$

   d) $\int \sqrt{x} \sin(1 + x^{3/2}) dx$

5. Find the area between the curves

   a) $y = |x|$, $y = x^2 - 2$

   b) $x + y = 0$, $x = y^2 + 3y$

6. Find the volume of a solid obtained by rotating the region bounded by

   a) $y = 1/x$, $y = 0$, $x = 1$, $x = 3$ about the line $y = -1$

   b) $y = x^2 - 3x + 2$, $y = 0$ about y-axis

7. The base of a solid is the region bounded by the parabolas $y = x^2$ and $y = 2 - x^2$. Find the volume of the solid if the cross-sections perpendicular to the x-axis are squares with one side along the base.

8. A force of 30 N is required to maintain a spring stretched from its natural length of $12cm$ to a length $15cm$. How much work is done in stretching the spring from $12cm$ to $20cm$?

9. problem 21, page 402

10. Find the numbers b such that the average value of $f(x) = 2 + 6x - 3x^2$ on the interval $[0, b]$ is equal to 3.