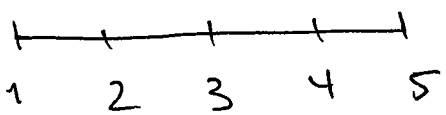
NAME: Solutions

Math 2423, Test I

September 28, 2005

Show all your work to receive full credit. The use of books and notes is not allowed. Good luck!

I. (20 pts) Approximate $\int_1^5 \frac{x-1}{x+1}dx$ with the Riemann sum R_4 with 4 terms, taking the sample points to be the right endpoints.



$$\Delta x = \frac{5-1}{4} = 1$$

$$R_4 = f(2)\cdot 1 + f(3)\cdot 1 + f(4)\cdot 1 + f(5)\cdot 1$$

$$= \frac{1}{3} + \frac{1}{2} + \frac{3}{5} + \frac{2}{3} = \boxed{\frac{21}{10}.}$$

II. (20 pts) Find the following definite and indefinite integrals

a) $\int \frac{\sqrt{u}-2u^2}{u} du$ $= \int (u^{-1/2} - 2u)\, du = 2u^{1/2} - u^2 + C$

b) $\int y^3\sqrt{2y^4 - 1}dy$ $= \frac{1}{8} \int \sqrt{u}\, du = \frac{1}{8} \cdot \frac{2}{3} u^{3/2} + C$

$u = 2y^4 - 1$

$du = 8y^3\, dy$ $= \frac{1}{12} (2y^4 - 1)^{3/2} + C$

$\frac{du}{8} = y^3\, dy$

c) $\int \frac{x^2}{\sqrt{1-x}} dx$ $= -\int \frac{(1-u)^2}{\sqrt{u}} du = -\int (u^{-1/2} - 2u^{1/2} + u^{3/2})\, du$

$u = 1 - x$ $= -2u^{1/2} + \frac{4}{3} u^{3/2} - \frac{2}{5} u^{5/2} + C$

$du = -dx$

$(1-u)^2 = x^2$ $= -2(1-x)^{1/2} + \frac{4}{3}(1-x)^{3/2} - \frac{2}{5}(1-x)^{5/2} + C$

d) $\int_1^5 \frac{dt}{(t-4)^2}$ $= \int_{-3}^{1} u^{-2}\, du = -u^{-1}\Big|_{-3}^{1} =$

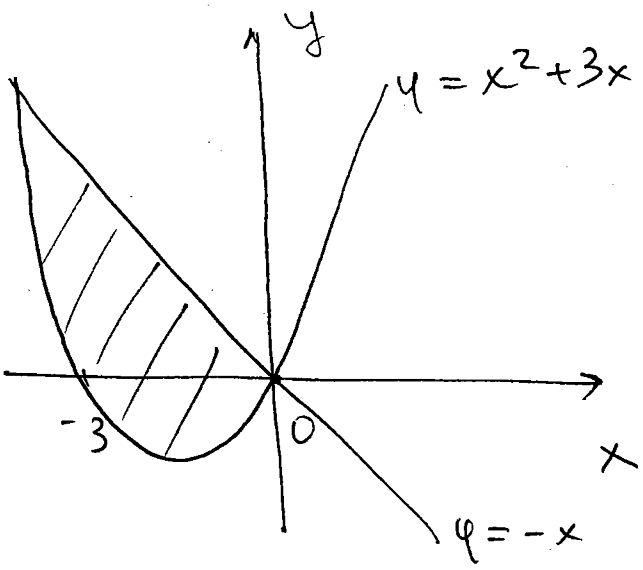
$u = t - 4$

$du = dt$ $= -1 + \left(-\frac{1}{3}\right) = -\frac{4}{3}$

$t = 1 \quad u = -3$

$t = 5 \quad u = 1$

III. (20 pts) Find the area of the region bounded by the curves $x+y=0$ and $y=x^2+3x$.



$$x^2+3x=-x$$

$$x^2+4x=0$$

$$x=0, \quad x=-4.$$

x-coordinates of points of intersection of two curves

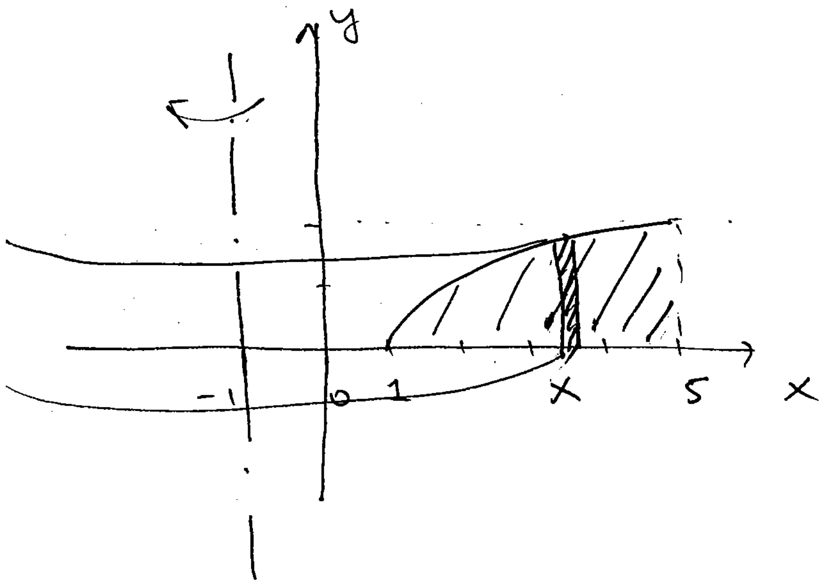
$$A=\int_{-4}^{0} -x-x^2-3x\,dx = \int_{-4}^{0}(-x^2-4x)\,dx$$

$$= -\frac{x^3}{3}-2x^2\Big|_{-4}^{0} =$$

$$= \cancel{\frac{64}{3}+} \quad 0 + \frac{(-4)^3}{3}+2(-4)^2$$

$$= -\frac{64}{3}+32 = \boxed{\frac{32}{3}}$$

IV. (20 pts) Find the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x-1}$, $x = 5$ and $y = 0$ about the line $x = -1$.



Cylindrical shells:

$$V = \int_1^5 2\pi r \cdot h \cdot dx$$

$r = x+1$

$h = \sqrt{x-1}$

$$V = \int_1^5 2\pi (x+1)\sqrt{x-1}\,dx$$

$u = x-1 \qquad x=1 \quad u=0$

$du = dx \qquad x=5, \quad u=4.$

$x+1 = u+2.$

$$V = \int_0^4 2\pi (u+2)\sqrt{u}\,du =$$

$$= 2\pi\left(\frac{2}{5}u^{5/2} + \frac{4}{3}u^{3/2}\right)\Big|_0^4 = 2\pi \cdot \frac{352}{15}.$$

Washers

$$V = \int_0^2 A(y)\,dy$$

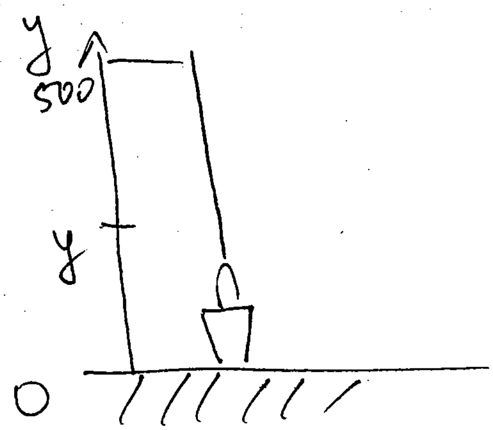
$A(y) = \pi\left(r_{out}^2 - r_{in}^2\right)$

$r_{out} = 5+1$

$r_{in} = y^2+1+1$

$$V = \pi\int_0^2 36 - (y^2+2)^2\,dy$$

V. (20 pts) A cable, 1 ft of which weighs 2 lb, is used to lift 800 lb of coal up a mine shaft 500 ft deep. Find the work done.



Work = $\int$ Force dy

Force = weight of bucket and cable at height y.

$F(y) = 800 + (500-y)\cdot 2$

$= 1800 - 2y$.

$$W = \int_0^{500} 1800 - 2y \, dy = 1800y - y^2 \Big|_0^{500}$$

$= 650000$ ft·lb.