

MATH2423-001 Project II: Drug Dosage

1 Problem Statement

The concentration in the blood resulting from a single dose of a drug normally decreases with time as the drug is eliminated from the body. In order to determine the exact pattern that the decrease follows, experiments are performed in which drug concentrations in the blood are measured at various times after the drug is administered. The data are then checked against a hypothesized function relating drug concentration to time. (We will make a simplifying assumption that when the drug is administered it reaches its fullest concentration instantaneously.)

Suppose a single dose of a certain drug is administered to a patient at time $t = 0$, and that the blood concentration is measured immediately thereafter, and again after four hours. The results of two such experiments are given in the table below.

Time	Experiment 1	Experiment 2
$t = 0$	1.0 mg/ml	1.5 mg/ml
$t = 4$	0.15 mg/ml	0.75 mg/ml

2 What to do

2.1 Part A

For this part, assume that the function describing concentration as a function of time is linear. Each data set in the table represents a different drug and a different initial dose. For each data set:

1. Sketch a graph of the concentration function, that is, graph the level of concentration vs. time. Assume concentrations are measured in milligrams per milliliter, and time is measured in hours.
2. Predict the time when the blood becomes free of the drug, assuming no further doses are administered.

3. Describe the rate at which the drug is eliminated.
4. Predict what the graph of concentration level vs. time would look like if further doses of the drug were administered every 6 hours for 48 hours.
5. Predict what would happen to the concentration level of the drug if it were administered every 6 hours indefinitely.

2.2 Part B

Now assume that the rate at which the concentration is decreasing at time t is proportional to the concentration level at time t . This idea can be modelled by a differential equation

$$\frac{dy}{dt} = -ky$$

where y is the concentration of the drug in the blood at time t , and k is a constant. The solution of the differential equation above is

$$y(t) = y_0 e^{-kt}.$$

Using the same data sets as in Part A, answer questions 1 to 5 above. (In fact, this model has been shown in clinical tests to be more accurate.)

2.3 Part C

A problem facing physicians is the fact that, for most drugs, there is concentration below which the drug will be ineffective and a concentration above which the drug will be dangerous.

Suppose that for the drug in Experiment 2, the minimum effective level is 0.45 mg/ml and the maximum safe level is 2.15 mg/ml. If the dose in the experiment is given every six hours, will the appropriate concentrations be maintained? Indefinitely? Explain. If the answer is NO, can you achieve a satisfactory long-run level just by adjusting the time between doses? Just by adjusting the dose?