Review Problems for Test I

Honors Calculus I

1) Let $f(x) = 1/x$, $g(x) = x^3$ and $h(x) = x^2 + 2$. Find $f \circ g \circ h$.

2) Express $H(x) = \sin^4(\sqrt{x})$ in the form $f \circ g \circ h$.

3) Determine whether the following limits exist. If yes, evaluate.

1. $\lim_{t\to 2} \frac{t^2+t-6}{t^2-4}$;
2. $\lim_{h\to 0} \frac{(1+h)^4-1}{h}$;
3. $\lim_{x\to 2} \frac{|x-2|}{x-2}$;
4. $\lim_{x\to 0} \sqrt{x^2+x^3} \sin \frac{2\pi}{x^3}$.

4) Prove each of the following statements using the ϵ - δ definition of a limit.

1. $\lim_{x\to 2}(4x + 11) = 19$;
2. $\lim_{x\to 0} x^3 = 0$.

5) Find the points at which $f(x)$ is discontinuous

$$f(x) = \begin{cases} 2x - 2 & \text{if} x \leq -1 \\ 3x & \text{if} -1 < x < 1 \\ 2x + 1 & \text{if} x \geq 1 \end{cases}$$

6) Find the constant c that makes $g(x)$ continuous on $(-\infty, +\infty)$

$$g(x) = \begin{cases} x^2 - c^2 & \text{if} x < 4 \\ cx + 20 & \text{if} x \geq 4 \end{cases}$$

7) Prove that the equation

$$x^5 - x^2 + 2x + 3 = 0$$

has at least one real root.

8)Find an equation of the tangent line to the curve at a given point.

1. $y = 1/\sqrt{x}$, at $P(4, 1/2)$;
2. $y = x^2 + 1$, at $P(1, 2)$.