

# Review for the Final

MATH 1823-001, Fall 2002

1. Calculate the limits:

a)  $\lim_{x \rightarrow 2} \frac{\sqrt[3]{x} - \sqrt[3]{2}}{x - 2};$

b)  $\lim_{x \rightarrow \infty} \frac{x^7 + 6x^3 + 11}{3x^7 - x^6 + x^5};$

c)  $\lim_{x \rightarrow 0} \frac{\tan 5x}{\tan 3x};$

d)  $\lim_{x \rightarrow 1} \frac{\sin(x-1)}{x^2 + x - 2};$

e)  $\lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{\sin \theta}.$

2. Find an equation of the tangent line to the curve  $y^2 = x^3(2 - x)$  at  $(1, 1)$ .

3. Show that the graphs of the equations  $x^2 - y^2 = 5$  and  $4x^2 + 9y^2 = 72$  intersect at right angles.

4. The angle of elevation of the sun is decreasing at a rate of  $0.25 \text{ rad/h}$ . How fast is the shadow cast by a 400-foot tall building increasing when the angle of elevation of the sun is  $\pi/6$ ?

5. Find the linearization of  $f(x) = \sqrt[3]{1 + 3x}$  at  $a = 0$ . Use it to give an approximate value for  $\sqrt[3]{1.03}$ .

6. Show that

$$\sqrt{1+x} < 1 + 1/2x$$

if  $x > 0$ .

7. Sketch the graph of the function that satisfies all of the given conditions

- $\lim_{x \rightarrow 3} f(x) = -\infty;$
- $f''(x) < 0$ , if  $x \neq 3;$

- $f'(0) = 0$ ;
- $f'(x) > 0$ , if  $x > 3$  or  $x < 0$ ;
- $f'(x) < 0$ , if  $0 < x < 3$ .

8. A conical drinking cup is made from a circular piece of paper of radius  $R$  by cutting out a sector and joining the edges. Find the maximum capacity of such a cup.

9. Find an equation of the line through the point  $(3, 5)$  that cuts off the least area from the first quadrant.

10. Find the local and absolute extreme values of the function on the given interval:

**a)**  $f(x) = \sqrt{x^2 + 4x + 8}$ , on  $[-3, 0]$ ;

**b)**  $f(x) = x - \sqrt{2} \sin x$ , on  $[0, \pi]$ .

11. Show that the equation  $x^{101} + x^{51} + x - 1 = 0$  has exactly one real root.

12. Find the point on the hyperbola  $xy = 8$  that is closest to the point  $(3, 0)$ .

13. Find  $f(x)$ , if  $f''(x) = x^4 - 4x^2 + 3x - 2$ , and  $f(0) = 0$ ,  $f(1) = 1$ .

14. A canister is dropped from a helicopter 500 m above the ground. Its parachute does not open, but the canister has been designed to withstand an impact velocity of 100 m/sec. Will it burst or not?