Let $f : \mathbb{R} \to [0, \infty)$ be a function that satisfies the property that for all x, y,

$$f(x + y) = f(x)f(y).$$

1. Prove that for all $n \in \mathbb{N}$ that $f(nx) = [f(x)]^n$.
2. Prove that if there is a single value x for which $f(x) \neq 0$, then $f(0) = 1$.
3. Prove for all $k \in \mathbb{Z}$ that $f(kx) = [f(x)]^k$.
4. Prove for all rational values $r \in \mathbb{Q}$ that $f(rx) = [f(x)]^r$. Hint: Start with $r = \frac{1}{k}$.
5. Prove that f is continuous at 0 if and only if f is continuous on $\mathbb{R}$.
6. Prove that if $f(x)$ is continuous at 0, then there is some positive real number a such that

$$f(x) = a^x$$

for all $x \in \mathbb{R}$.