Inverse Functions

Exploring One to One Functions

1. Graph $f(x) = x^2$
2. This function is not one-to-one. We can see this if we draw the **horizontal** line $y = 1$, which intersects the graph more than once! This is called the horizontal line test. Why does this stop $f(x) = x^2$ from having an inverse function?
3. Graph $g(x) = \dfrac{1}{1-x}$
4. Is $g(x)$ one-to-one? If so, find the inverse function.

Finding Inverse Functions

Find the inverse function of the following:

1. $f(x) = 1 + \sqrt{2+3x}$
2. $y = \dfrac{4x-1}{2x+3}$
3. $y = x^2 - x$, $x \geq 2$
   Why do we need to restrict the domain here?

Derivatives of Inverse Functions

We should memorize the formula:

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

Use this formula to find the derivative of the f^{-1}:

1. $f(x) - \sqrt{4x+3}$
2. $f(x) = 2 - x^4$, $x \geq 0$
3. $f(x) = 3x^3 + 4x^2 + 6x + 5$ at $x = 5$ This is the derivative at the value $x = 5$, so your solution should be a number.
4. $f(x) = 3 + x^2 + \tan(\frac{\pi x}{2})$, $-1 < x < 1$ at $x = 3$

Logarithms

Three Important Rules

- $\log_a(xy) = \log_a x + \log_a y$
- $\log_a(\frac{x}{y}) = \log_a x - \log_b y$
- $\log_a(x^r) = r \log_a x$

Expand the following:

1. $\ln \sqrt{ab}$
2. $\ln \left(\dfrac{x^2}{y^2 z^4} \right)$
3. $\ln(x + y)$ Be very careful here!

How do we evaluate a logarithm?

When we see $\log_2 8$, it is asking for the power of 2 that gives 8. Thus, $\log_2 8 = 3$ because $2^3 = 8$.

Try: $\log_3 81$ and $\ln e^7$. What about $\ln 108$?

Differentiation

Differentiate the following:

1. $\ln x$
2. $\ln(\sin x)$
3. $x^3 \ln x$

Integration

Evaluate the following integrals:

1. $\int \dfrac{3}{x}\, dx$
2. $\int (\sqrt{x} + \frac{1}{\sqrt{x}})^2\, dx$
3. $\int \dfrac{x^2 + x + 1}{x}\, dx$
4. $\int \dfrac{\cos x}{2 + \sin x}\, dx$