Week 2 worksheet
(due Monday, June 19 at the beginning of class)

1. (a) Show that the following set of vectors in \( M_{22} \) is linearly dependent:
\[
\begin{bmatrix}
1 & 0 \\
1 & 1
\end{bmatrix}, \quad
\begin{bmatrix}
0 & 2 \\
1 & -1
\end{bmatrix}, \quad
\begin{bmatrix}
3 & -4 \\
1 & 5
\end{bmatrix}, \quad
\begin{bmatrix}
1 & 1 \\
0 & 2
\end{bmatrix}.
\]

(b) Express one vector as a linear combination of the others.

2. Find a basis for \( \mathbb{R}^4 \) containing the following vectors:
\[
\begin{bmatrix}
1 \\
1 \\
0 \\
1
\end{bmatrix}, \quad
\begin{bmatrix}
0 \\
1 \\
1 \\
0
\end{bmatrix}, \quad
\begin{bmatrix}
0 \\
0 \\
1 \\
-1
\end{bmatrix}.
\]

3. Find a spanning set for the subspace of \( P_2 \) consisting of polynomials \( at^2 + bt + c \), where \( a + 2b = c \).

4. Let \( S = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\} \) and \( T = \left\{ \begin{bmatrix} 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \end{bmatrix} \right\} \) be ordered bases for \( \mathbb{R}^2 \). Let \( \mathbf{v} = \begin{bmatrix} 4 \\ 4 \end{bmatrix} \).

(a) Find the coordinate vector for \( \mathbf{v} \) with respect to the basis \( T \).

(b) Find the transition matrix \( P \) from the \( T \)-basis to the \( S \)-basis.

(c) Use your answer to (b) to find the coordinate vector for \( \mathbf{v} \) with respect to \( S \).

5. (a) Let \( S = \{\mathbf{v}_1, \mathbf{v}_2\} \) and \( T = \{t + 2, 1\} \) be ordered bases for \( P_1 \). Find the basis \( S \) using the fact that the transition matrix from \( S \) to \( T \) is \( \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \).

(b) Let \( S = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\} \) and \( T = \{\mathbf{w}_1, \mathbf{w}_2\} \) be ordered bases for \( \mathbb{R}^2 \). Find the basis \( T \), using the fact that the transition matrix from \( S \) to \( T \) is \( \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} \).