Name:  
OUID:  

**Instructions:** Be sure to show as much work as possible, and please make a sincere effort to express your answers clearly and neatly. Please write your answers on your own paper, then staple your pages together using this sheet as a cover sheet.

1. Consider the following bases for $\mathbb{R}^2$ and $\mathbb{R}^3$:

   $$S = \left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\} \quad S' = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$$

   $$T = \left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \end{bmatrix} \right\} \quad T' = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \end{bmatrix} \right\}$$

and note that the transition matrices are given by

$$P_{S\rightarrow S'} = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix} \quad Q_{T\rightarrow T'} = \begin{bmatrix} 0 & 0 & -1 \\ 1 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix}.$$  

Let $L : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be a linear map whose matrix representation with respect to $S$ and $T$ is $A = \begin{bmatrix} -1 & -1 \\ 0 & 3 \\ 2 & -1 \end{bmatrix}$, and let $[v]_{S'} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$.

(a) [3 pts] Find the inverse of $Q$.

(b) [3 pts] Use the matrices above and your answer to part (a) to find the matrix representation $B$ for $L$ with respect to $S'$ and $T'$.

(c) [3 pts] Use matrix multiplication to find $[v]_S$, $[L(v)]_T$ and $[L(v)]_{T'}$.

(d) [3 pts] Find $v$ and $L(v)$.

(e) [6 pts] Find the matrix representation for $L$ with respect to the standard bases for $\mathbb{R}^2$ and $\mathbb{R}^3$.

2. [3 pts] Let $V$ be the set of all ordered pairs $(a, b)$ of real numbers with the operations

$$(a, b) \oplus (c, d) = (a + c, b + d) \quad r \odot (a, b) = (a, rb).$$

Show that this set with these operations is *not* a vector space.