Linear Algebra I Worksheet 3 Due June 22, 2007

Name: OUID:

Instructions: Be sure to show as much work as possible, and please make a sincere effort to express your answers clearly and neatly. Please write your answers on your own paper, then staple your pages together using this sheet as a cover sheet.

1. Consider the linear map \( L : \mathbb{R}^4 \to \mathbb{R}^4 \) given by
   \[
   L \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} b - d \\ a + b + c \\ -a + 4d \\ 3b + c - d \end{pmatrix}.
   \]
   
   (a) [3 pts] Find the standard matrix representation \( A \) for \( L \).
   
   (b) [4 pts] Show that \( L \) is an isomorphism by finding the inverse of the matrix \( A \) from part (a).
   
   (c) [3 pts] Use your answer to part (b) to write a formula for \( L^{-1} : \mathbb{R}^4 \to \mathbb{R}^4 \) like that for \( L \) above.

2. Consider the basis \( S = \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \right\} \) for \( \mathbb{R}^3 \).
   
   (a) [3 pts] Suppose \( [v]_S = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \). Find \( v \).
   
   (b) [3 pts] Find the \( S \)-coordinates for \( \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \).

3. [5 pts] Show that \( [v]_S + [w]_S = [v + w]_S \) for any two vectors \( v \) and \( w \) and any basis \( S \).