Putnam Problems – "Now for Something Completely Different"

1. If $A, B \in M_n(\mathbb{C})$, $A \neq B$, $A^3 = B^3$, and $A^2B = B^2A$, is $A^2 + B^2$ necessarily singular?

2. Do there exist polynomials $a(x)$, $b(x)$, $c(y)$, $d(y)$ such that $1 + xy + x^2y^2 = a(x)c(y) + b(x)d(y)$?

3. Does there exist a totally ordered, uncountable chain of subsets of $\mathbb{Z}$?

4. Is it possible to place weights on two six-sided dice in such a way that the probability of getting any particular sum from 2 to 12 is $\frac{1}{11}$?

5. Consider the grid $\mathbb{Z} \times \mathbb{Z}$. You are standing at the origin O. You can see a point P if no other point on the segment OP lies on the grid; otherwise P is blocked. Do there exist arbitrarily large squares of blocked points?