Putnam Seminar — Week 3 — Sept. 30, 2015

1. (This is a Putnam problem, but I'm not sure from what year:)
Consider the sequence u_n defined by $u_0 = u_1 = u_2 = 1$, and

$$\det \begin{pmatrix} u_{n+3} & u_{n+2} \\ u_{n+1} & u_n \end{pmatrix} = n!,$$

for $n \geq 0$. Prove that u_n is an integer for all n.

2. (Not actually a Putnam problem:)
Define

$$x_n = \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \cdots + \sqrt{1}}}}},$$

where there are n 1's on the right-hand side. Show that the sequence x_n converges to a limit, and find the limit.