More problems from *Problem-solving Through Problems* by Loren C. Larson.

5. Determine all solutions in real numbers of the system

$$\begin{aligned} x + y + z &= w \\ \frac{1}{x} + \frac{1}{y} + \frac{1}{z} &= \frac{1}{w}. \end{aligned}$$

6. Prove that there are no prime numbers in the infinite sequence of integers

$$10001, 100010001, 1000100010001, \ldots.$$

7. Prove that any two numbers of the following sequence are relatively prime:

$$2+1, 2^2+1, 2^4+1, 2^8+1, \ldots, 2^{2^n}+1, \ldots.$$

Show that this results proves that there are an infinite number of primes.

8. Let f be a polynomial with real coefficients. Show that all the zeros of f are real if and only if f^2 cannot be written as the sum of squares

$$f^2 = g^2 + h^2$$

where g and h are polynomials with real coefficients and the degree of g is not equal to the degree of h.