

Here are some relatively easy, but fun and instructive problems from *Problem-solving through problems* by Loren C. Larson, Springer-Verlag 1983. You can read this book online at <https://math.la.asu.edu/~ifulman/mat194/problem-solving.pdf> .

1. A chord of constant length slides around in a semicircle. The midpoint of the chord and the projections of its ends upon the base form the vertices of a triangle. Prove that the triangle is isosceles and never changes its shape.

2. If a and b are positive integers with no common factor, show that

$$\left\lfloor \frac{a}{b} \right\rfloor + \left\lfloor \frac{2a}{b} \right\rfloor + \left\lfloor \frac{3a}{b} \right\rfloor + \cdots + \left\lfloor \frac{b-1}{b} \right\rfloor = \frac{(a-1)(b-1)}{2}.$$

(Here $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x .)

3. A rectangular room measures 30 feet in length and 12 feet in height, and the ends are 12 feet in width. A fly, with a broken wing, rests at a point one foot down from the ceiling at the middle of one end. A smudge of food is located one foot up from the floor at the middle of the other end. The fly has just enough energy to *walk* 40 feet. Show that there is a path along which the fly can walk that will enable it to get to the food.

4. Let a and b be given positive real numbers with $a < b$. If two points are selected at random from a straight line segment of length b , what is the probability that the distance between them is at least a ?