**Putnam Seminar — Week 4**

**17.** Find the length of the longest sequence of equal non-zero digits in which an integral square can terminate (in base 10) and find the smallest square which terminates in such a sequence.

**18.** Given a sequence $\{x_n\}$, $n = 1, 2, 3, \ldots$, such that $\lim_{n\to\infty}(x_n - x_{n-2}) = 0$, prove that

$$\lim_{n\to\infty} \frac{x_n - x_{n-1}}{n} = 0.$$

**19.** A closed subset $S$ of $\mathbf{R}^2$ lies in the set $\{(x, y) \in \mathbf{R}^2 : a < x < b\}$. Show that its projection on the $y$-axis is closed.

**20.** Three numbers are chosen independently at random, one from each of the the three intervals $[0, L_i]$, $(i = 1, 2, 3)$. If the distribution of each random number is uniform with respect to length in the interval it is chosen from, determine the expected value of the smallest of the three numbers chosen.