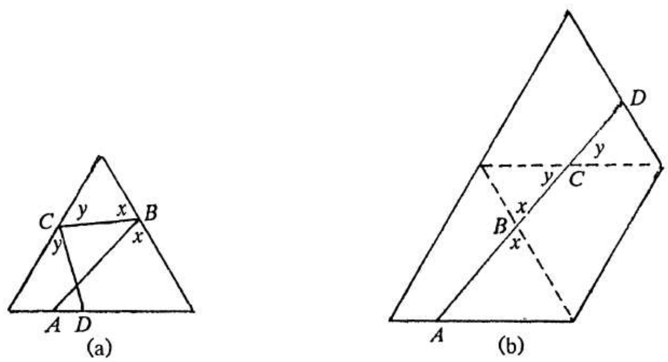
Light rays can be equated to billiard paths in such a way that a light source is simply a set of infinitely many rays emitting from one source. These rays are like billiard paths in that a billiard path itself is a ray. A light ray, when in contact with a mirrored surface reflects off the surface with the same angle it met the surface. (The angle of incidence = the angle of reflection) When a billiard ball come in contact with a wall, it bounces off the wall with the same property as the ray.

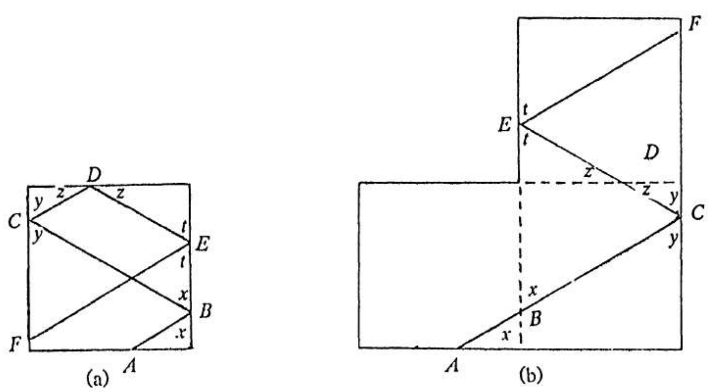
In order for the proof by Tokarsky to work, me must make some assumptions. First of all, when a light ray, or billiard path (the two will be used in conjunction here strikes a vertex, it is absorbed there since it has nowhere else to go, also all pool shots must be taken of non-zero length.

One of the main ideas of this proof is that when taking a billiard shot inside of a polygon, you can unfold mirror images of your original polygon to portray another version of the billiard path, you may then fold the unfolded path back up to receive the original view.

In the the following example, a pool shot is taken inside a triangle. The first collision is at point B, with angle x, then at C with angle y and winding up at point D.

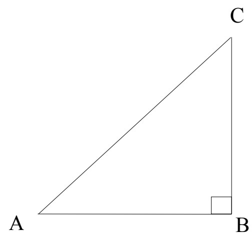


The reason that the angles must be the same is due to the theorem that opposite angles have equal measures. In this example, x=x and y=y. In previous figure (b) was formed by unfolding (a) three times to form a straight line path from A to D. This unfolding could have been achieved in a different way, again providing an alternative to the original view of the path. This same property of unfolding may be seen the the next figure as well.

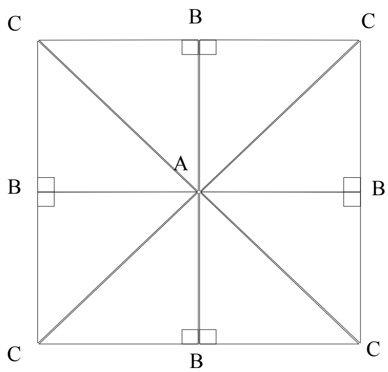


The preceding picture again shows how a billiard path, this time in a square figure from A to E, can be unfolded to convey another view of the same path.

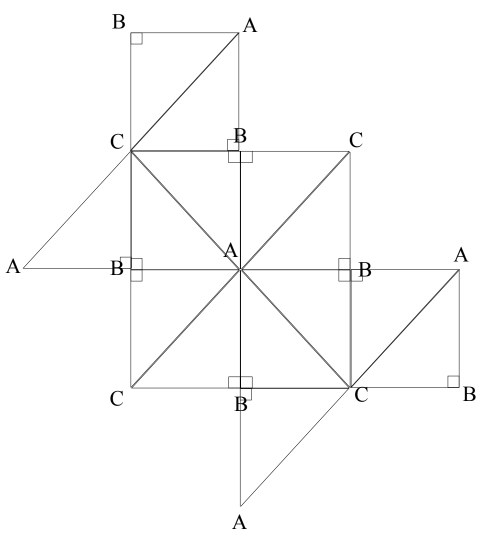
In order to construct a room in such a way that at least on point in a room where there exists one light source is unilluminable, the key is to use the preceding lemma and all of the points labeled B and C must lie on vertices of the room. The room is defined as illuminated if each pint of the room lies on at least one of the source's rays. The light source may be placed anywhere in the room. To create this unilluminable room, we start off with a right triangle, with vertices labeled A, B, C.



We will unfold mirror images of this triangle until we have achieved our unilluminable room.

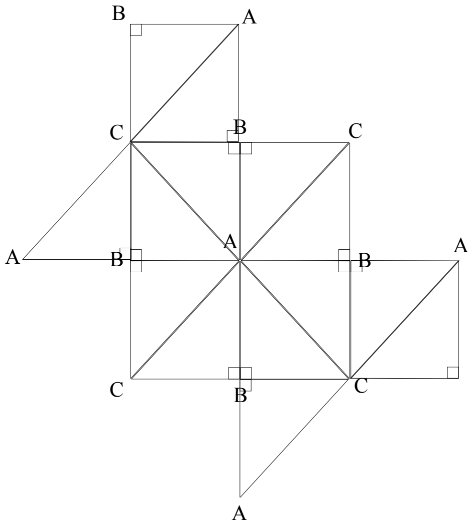


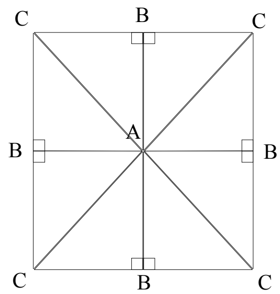
First we unfold them into eight triangles, forming a square. Now if we take a billiard shot from our original A, it will only be absorbed by our four C vertices's. We must now fix any other possible shots.



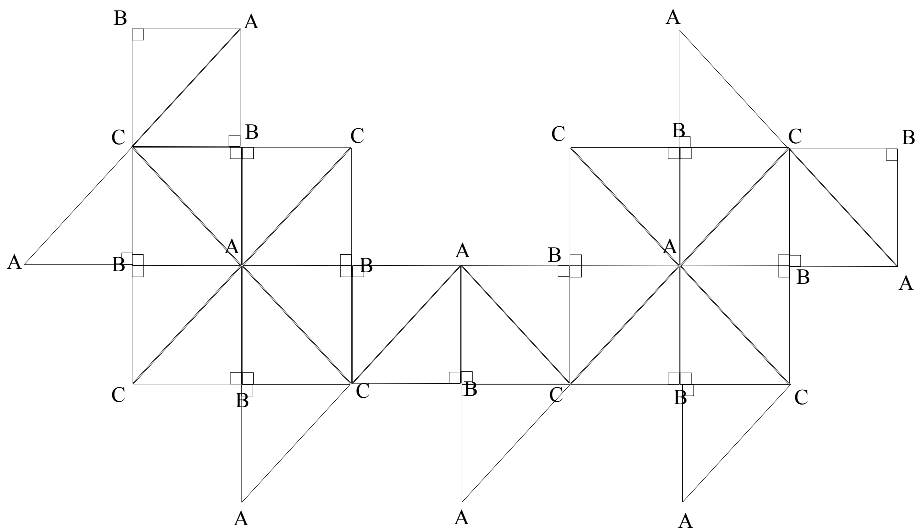
Now no matter which line you follow emitting from A that corresponds with a side of the original triangle, you will hit a vertex, thus being absorbed. We need this to be the case because in our lemma we proved that no shot in a triangle, when taken from A can come back to A. In that lemma, you obviously can not traverse a side since the side is a wall, in this example, we accommodate for this by wherever their should be a wall (an impossible shot in our lemma) where a shot would go, the shot will hit a vertex, thus being absorbed there.

Now that we have a point A in the middle of the room, from this point A, it is impossible for a billiard shot to contact another point A. However, since all of the A's are on a vertex or wall, there is no other point A in the room that is not illuminable. So let's now create another point in the room...





Now we have two points A in the middle of the room. Let's now connects the two rooms using the same rules as before.



Now we have constructed a room with two points A in the middle of the room (not on a wall or a vertex). Since the room has been formed in such a way that the room is simply a lattice of unfolded triangles that are mirror images of each other, the lemma proved earlier still holds for this room.
If a ray emits from any point A, it must either follow a former "side" of the original triangle, which, in our room, leads to striking a vertex and being absorbed, or it must enter one of the eight surrounding triangles which we have shown may be folded up to make the original shot from A to A in a single triangle which by the lemma was proved impossible.

The room constructed in this paper, was the smallest Tokarksy, or anyone else so far, has been able to find, twenty-six sides. Tokarsky has proved that a room in two-dimensions can be not be illuminated from a finite number of points, however the question remains open in three dimensions as well as the questions as to whether or not there exists a room so complex that there are infinitely many points in the room that do not illuminate the room. These interesting questions are left to the rest of us to answer...

References

Tokarsky, George. Polygonal Rooms Not Illuminable from Every Point. The American Mathematical Monthly, Vol 102, No 10 (Dec 1995) 867-879.