

Quiz 6

Name _____

Row _____

1. Evaluate the integral $\int \sin^3 \theta \cos^3 \theta d\theta$.

$\textcircled{1} u = \sin \theta$
 $\textcircled{1} du = \cos \theta d\theta$

$$\begin{aligned}
 &= \int \sin^3 \theta \cos^2 \theta \cos \theta d\theta \\
 &= \int u^3 \cos^2 \theta du \\
 &= \int u^3 (1 - \sin^2 \theta) du \\
 &= \int u^3 (1 - u^2) du \\
 &= \int u^3 - u^5 du = \frac{u^4}{4} - \frac{u^6}{6} + C
 \end{aligned}$$

$$= \frac{\sin^4 \theta}{4} - \frac{\sin^6 \theta}{6} + C$$

2. Evaluate the integral $\int \sqrt{x^2 - 1} dx$. Try to simplify your answer as much as possible. You may use the formulas

$$\int \sec \theta d\theta = \ln |\sec \theta + \tan \theta|$$

$$\int \sec^3 \theta d\theta = \frac{1}{2} (\ln |\sec \theta + \tan \theta| + \sec \theta \tan \theta)$$

$x = \sec \theta$

$dx = \sec \theta \tan \theta d\theta$

$$\int \sqrt{\sec^2 \theta - 1} \sec \theta \tan \theta d\theta$$

$$= \int \sqrt{\tan^2 \theta} \sec \theta \tan \theta d\theta =$$

$$= \int \sec \theta \tan^2 \theta d\theta = \int \sec \theta (\sec^2 \theta - 1) d\theta$$

$$= \int \sec^3 \theta d\theta - \int \sec \theta d\theta =$$

$$= \frac{1}{2} (\ln |\sec \theta + \tan \theta| + \sec \theta \tan \theta) - \ln |\sec \theta + \tan \theta|$$

$$= \frac{1}{2} \ln |\sec \theta + \tan \theta| + \frac{1}{2} \sec \theta \tan \theta = \frac{1}{2} \ln |x + \sqrt{x^2 - 1}| + \frac{1}{2} x \sqrt{x^2 - 1}$$

